

Characteristic of Bipolar Fuzzy and Bipolar Fuzzy Divisor of Zero of a Ring

M. Rajeshwari¹, K. A. Venkatesh²

Department of Mathematics, School of Engineering, Presidency University, Bangalore, India

Department of Mathematics and Comp. Science, School of Engineering, Presidency University, Bangalore, India.

E-mail: rajeakila@gmail.com, prof.kavenkatesh@gmail.com

Abstract - During this paper, we present the documentation of characteristic of bipolar fuzzy and bipolar fuzzy divisor of zero of a ring, and study essential properties. We concentrate fundamental operation on characteristic of bipolar fuzzy and bipolar fuzzy divisor of a ring and explore a few properties of these ideas in association with bipolar fuzzy perfect of a ring.

Keywords. – Characteristic Bipolar fuzzy, bipolar fuzzy divisor of zero of a ring, bipolar fuzzy order of an element, bipolar fuzzy homomorphism of ring, Product of bipolar fuzzy subsets.

I. INTRODUCTION

Fuzzy set was introduced in 1965 by Zadeh [1], technique for speaking to unclearness and vulnerability. The hypothesis of fuzzy set has turned into an energetic range of examine in various controls. The idea of Bipolar fuzzy set was presented in 1994 by Zhang [2], as a speculation of fuzzy set. A Bipolar fuzzy set is the expansion of fuzzy set whose enrollment degree run in $[-1,1]$. It's a pair of fuzzy sets, as membership and non-membership function, which represent the positive and negative aspects of the given data. In this paper we have presented the idea of bipolar fuzzy characteristic and bipolar fuzzy divisor of zero of ring. By utilizing the idea we have demonstrated that some essential properties of in the ring theory identifying with fuzzy characteristic, fuzzy divisor of zero and orders of in additive group of a ring.

II. CHARACTERISTIC OF BIPOLAR FUZZY AND BIPOLAR FUZZY DIVISOR OF ZERO

Some basic definitions

Definition: 2.1. Let G be a group and A be a bipolar fuzzy subgroup of a group G with the identity e . For each $x \in G$, the the smallest amount positive whole number n , such that $\mu_A(x^n) = [\mu_A^P(e), \mu_A^N(e)]$ specified is named Bipolar fuzzy order of x with relevance to $A[\mu(FO_A(x))]$. If no such n exists, x is claimed to possess infinite bipolar fuzzy order with respect to A .

Definition: 2.2. . Let A be a Bipolar fuzzy subring of a hoop R . the smallest amount positive whole number m , satisfies $\mu_A(mx) = [\mu_A^P(o), \mu_A^N(o)]$ for all x in R said to be Bipolar fuzzy characteristic of the ring R with relation to $A[\mu(FC_A(x))]$, wherever o could be a zero of the ring. R is said to be Bipolar fuzzy characteristic zero (or infinity) with relation to A iff $m=0$ is that the solely whole number such that $\mu_A(mx) = [\mu_A^P(o), \mu_A^N(o)]$ for all x in R .

Definition: 2.3. [4] A fuzzy subset of a ring R is called a fuzzy ideal of R iff for all $x, y \in R$,

$$i) A(x - y) \geq \min(A(x), A(y))$$

$$ii) A(xy) \geq \max(A(x), A(y)).$$

Definition: 2.4. Let A be a bipolar fuzzy subring or a ring R .

Promote, let $a, b \in R$ such that $\mu_A(ab) \neq [\mu_A^P(o), \mu_A^N(o)]$

and $\mu_A(a) \neq [\mu_A^P(o), \mu_A^N(o)], \mu_A(b) \neq [\mu_A^P(o), \mu_A^N(o)]$.

At that point a is known as a left bipolar fuzzy divisor of zero and b is known as a privilege bipolar fuzzy divisor of zero.

Definition: 2.5. Assume A be a bipolar fluffy regarding A . subring of a ring R with character e such that

$$0 \neq [\mu_A^P(e), \mu_A^N(e)] \neq [\mu_A^P(o), \mu_A^N(o)].$$

An component with $a \in R$ with $\mu_A(a) \neq [\mu_A^P(o), \mu_A^N(o)]$

is said to be a fuzzy unit of R concerning A quickly as

$\mu(FU_A(R))$ though there exist $a \in R$ with

$$\mu_A(b) \neq [\mu_A^P(o), \mu_A^N(o)] \text{ such that}$$

$$\mu_A^P(ab) = \mu_A^P(ba) = \mu_A^P(e) \text{ and}$$

$$\mu_A^N(ab) = \mu_A^N(ba) = \mu_A^N(e).$$

Definition: 2.6. [6] Let ' \bullet ' be a binary operation in a nonempty set X . A fuzzy subset P of X is said to be translational invariant with respect to ' \bullet ' if for any $x, y \in X$, $P(x) = P(y) \Rightarrow P(x \bullet a) = P(y \bullet a)$ and $P(a \bullet x) = P(a \bullet y)$ for all $a \in X$.

Theorem: 2.1. Let A be a bipolar fuzzy ideal of the ring R with associate identity e . Then $[\mu(FC_A(R))] = [\mu(FO_A(e))]$ is an associate additive group of R .

Proof: : Allow us to assume $[\mu(FO_A(e))]$ is limited equivalent to m . At that point we have $\mu_A^P(me) = \mu_A^P(o)$,

$$\mu_A^N(me) = \mu_A^N(o). \text{ Then we've Let } x \in R$$

$$\mu_A(mx) \geq \max(\mu_A^P((me)(x)), \mu_A^N((me)(x)))$$

Where

$$\mu_A^P((me)(x)) \geq \min(\mu_A^P(me), \mu_A^P(x))$$

$$= \min(\mu_A^P(o), \mu_A^P(x))$$

$$= \mu_A^P(o)$$

$$\begin{aligned}\mu_A^N((me)(x)) &\leq \max(\mu_A^N(me), \mu_A^N(x)) \\ &= \max(\mu_A^N(o), \mu_A^N(e)) \\ &= \mu_A^N(o)\end{aligned}$$

$$\therefore \mu_A^P((me)(x)) \geq \mu_A^P(o) \text{ and } \mu_A^N((me)(x)) \leq \mu_A^N(o)$$

which infers $\mu_A(mx) = [\mu_A^P(o), \mu_A^N(o)]$ for all $x \in R$, $[\mu(FC_A(R))]$ is m.

On the other hand it's obvious that $[\mu(FC_A(R))]$ is zero, just in the event that $[\mu(FO_A(e))]$ is boundless.

Theorem: 2.2. Let A be a bipolar fuzzy ideal of a ring R with no fuzzy divisor of zero with relation to A and a, b \in R be such that

$$\begin{aligned}\mu_A(a) &\neq [\mu_A^P(o), \mu_A^N(o)] \text{ and } \\ \mu_A(b) &\neq [\mu_A^P(o), \mu_A^N(o)]. \text{ At the point } \\ [\mu(FO_A(a))] &= [\mu(FO_A(b))] \text{ in added substance gathering of R.}\end{aligned}$$

Proof: Let us accept that $[\mu(FO_A(a))]$ is limited and equivalent to m. It is adequate to demonstrate that for any whole number m, such that $\mu_A^P(mb) = \mu_A^P(o)$,

$$\mu_A^N(mb) = \mu_A^N(o).$$

We've,

$$\begin{aligned}\mu_A(a(mb)) &= \mu_A(m(ab)) \\ &= \mu_A((ma)b) \\ &\geq \max(\mu_A^P((ma)(b)), \mu_A^N((ma)(b)))\end{aligned}$$

where

$$\begin{aligned}\mu_A^P((ma)(b)) &\geq \min(\mu_A^P(ma), \mu_A^P(b)) \\ &= \min(\mu_A^P(o), \mu_A^P(b)) \\ &= \mu_A^P(o)\end{aligned}$$

$$\begin{aligned}\mu_A^N((ma)(b)) &\leq \max(\mu_A^N(mb), \mu_A^N(a)) \\ &= \max(\mu_A^N(o), \mu_A^N(a)) \\ &= \mu_A^N(o)\end{aligned}$$

$$\text{shows that } \mu_A(a(mb)) = [\mu_A^P(o), \mu_A^N(o)].$$

Since R has no bipolar fuzzy divisor of zero with concerning to A and $\mu_A(a) \neq [\mu_A^P(o), \mu_A^N(o)] \Rightarrow \mu_A(mb) = [\mu_A^P(o), \mu_A^N(o)]$.

On the opposite hand, if $\mu_A(nb) = [\mu_A^P(o), \mu_A^N(o)]$ for few positive number $n < m$, we discover

$$\begin{aligned}\mu_A((na)b) &= (a, b \in R / \mu_A^P((a)(nb)), \mu_A^N((a)(nb))) \\ &= (a, b \in R / \mu_A^P((a)(nb)), \mu_A^N((a)(nb))) \\ &= (a, b \in R / \mu_A^P(o), \mu_A^N(o))\end{aligned}$$

which gives $\mu_A(na(b)) = [\mu_A^P(o), \mu_A^N(o)]$. Since R has no bipolar fuzzy divisor of zero with relation to A and $\mu_A(b) \neq [\mu_A^P(o), \mu_A^N(o)]$.

We should have $\mu_A(na) = [\mu_A^P(o), \mu_A^N(o)]$. This ends up in contradiction. Hence $\mu(FO_A(b)) = m$.

Theorem: 2.3. Let A be bipolar fuzzy ideal ring R with no Bipolar of zero with relevance to A and $\mu(FC_A(R)) \neq 0$. If $\mu_A(a) \neq [\mu_A^P(o), \mu_A^N(o)]$ for a few some $a \in R$, then $\mu(FC_A(R))$ be a prime.

Proof: Let $\mu(FC_A(R))$ be m and $\mu_A(a) \neq [\mu_A^P(o), \mu_A^N(o)]$ for $a \in R$. Then $\mu(FO_A(a))$ is m. Let m spoke to as the result of pq of two whole number p and q. At that point we have

$$\begin{aligned}\mu_A((pa)(qa)) &= \mu_A(pqa^2) \\ &= \mu_A((ma)a) \\ &\geq \max(\mu_A^P((ma)(a)), \mu_A^N((ma)(a)))\end{aligned}$$

where

$$\begin{aligned}\mu_A^P((ma)(a)) &\geq \min(\mu_A^P(ma), \mu_A^P(a)) \\ &= \min(\mu_A^P(o), \mu_A^P(a)) \\ &= \mu_A^P(o) \\ \mu_A^N((ma)(a)) &\leq \max(\mu_A^N(mb), \mu_A^N(a)) \\ &= \max(\mu_A^N(o), \mu_A^N(a)) \\ &= \mu_A^N(o)\end{aligned}$$

$$\therefore \mu_A((ma)(a)) \geq \mu_A^P(o) \text{ and } \mu_A^N((ma)(a)) \leq \mu_A^N(o).$$

Since R has no bipolar fuzzy divisor of zero with regarding to

A. We tend to should have $\mu_A(pa) = (\mu_A^P(o), \mu_A^N(o))$ or $\mu_A(qa) = (\mu_A^P(o), \mu_A^N(o))$.

Since $\mu(FO_A(a))$ is m, one of the two whole number p and q must be m and consequently other must be one. Thus m is prime.

Theorem.2.4. Assume A will be a bipolar fuzzy ideal of the ring R with identity e to such an extent that and $0 \neq [\mu_A^P(e), \mu_A^N(e)] \neq [\mu_A^P(o), \mu_A^N(o)]$ and A is translational invariant with relevance to multiplication. If the set of all components of R that don't seem to be

$\mu(FU_A(R))$ form associate subgroup of R, then $\mu(FU_A(R))$ is either zero instead an influence of prime.

Proof: Assume that the arrangement of all component of R which are not $\mu(FU_A(R))$ seem to be kind of additive subgroup of R. Suppose $\mu(FU_A(R)) = n$ not equivalent to zero and n has two particular prime divisors p, q. Presently $\mu_A(pe) \neq [\mu_A^P(o), \mu_A^N(o)]$ and

$\mu_A(qe) \neq [\mu_A^P(o), \mu_A^N(o)]$. we tend to claim pe is not a $\mu(FU_A(R))$. For, if pe is a $\mu(FU_A(R))$ then there exist $u \in R$ such that $\mu_A(u) \neq [\mu_A^P(o), \mu_A^N(o)]$ and $\mu_A(pe(u)) = \mu_A(e)$. Then

$\mu_A((mq)e(pe)u) = \mu_A((mq)e)e = \mu_A((mq)e)$ Since A is translational invariant concerning augmentation, where m could be a number such $n=mpq$. Then

$$\begin{aligned}\mu_A((mq)e) &= \mu_A((mpq)e)u \\ &= \mu_A((ne)u) \\ &\geq \max(\mu_A^P((ne)(u)), \mu_A^N((ne)(u)))\end{aligned}$$

where

$$\begin{aligned}\mu_A^P((ne)(u)) &\geq \min(\mu_A^P(ne), \mu_A^P(u)) \\ &= \min(\mu_A^P(o), \mu_A^P(u)) \\ &= \mu_A^P(o) \\ \mu_A^N((ne)(u)) &\leq \max(\mu_A^N(ne), \mu_A^N(u)) \\ &= \max(\mu_A^N(o), \mu_A^N(u)) \\ &= \mu_A^N(o)\end{aligned}$$

$$\therefore \mu_A^P((ne)(u)) \geq \mu_A^P(o) \text{ and } \mu_A^N((ne)(u)) \leq \mu_A^N(o).$$

This demonstrate $\mu_A((mq)e) = [\mu_A^P(o), \mu_A^N(o)]$ what is the disagreement since $mq < n$. Hence pe is not a $\mu(FU_A(R))$.

Consequently qe is not a $\mu(FU_A(R))$. since pe and qe are not $\mu(FU_A(R))$, any number direct blend of them is not $\mu(FU_A(R))$ by suspicion. Be that as it may, we have $ps+qt=1$ for some whole number s and t since p, q are coprime and henceforth, $[\mu_A^P(e), \mu_A^N(e)] \neq [\mu_A^P(o), \mu_A^N(o)]$. This demonstrate that $s(pe)+t(qe)$ is a $\mu(FU_A(R))$, which is the disagreement. therefore n should be power of prime.

III. CONCLUSIONS

During this paper, we have a tendency to outline characteristic of Bipolar fuzzy and bipolar fuzzy divisor of zero of a ring. The properties of those notions in reference to bipolar fuzzy ideal of a ring are derived.

References

- [1] Zadeh, L.A.(1965), Fuzzy sets, Inform. And Control, 8: 338-353.
- [2] Lee, K.M.(2000), Bipolar interval valued fuzzy sets and their operations. Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand, pp: 307-312.
- [3] Anitha.M.S, K.L.Muruganatha Prasad & K.Arjunan, Notes on Bipolar valued fuzzy subgroups of a group, Bulletin of Society for Mathematical Services and Standards, Vol. 2, No. 3, 52-59,2013.
- [4] Liu, W.-J., Fuzzy invariant subgroup and fuzzy ideals. Fuzzy Sets and Systems, 8 (1982) 133–139.
- [5] Novak, V., Fuzzy Sets and Their Applications. Bristol: Adam Hilger 1989.

[6] Ray, A. K., Quotient group of a group generated by a subgroup and a fuzzy subset. Journal of Fuzzy Mathematics, 7(2) (1999), 459–463.

[7] Asok Kumar Ray., A note on fuzzy characteristic and fuzzy divisor of zero of a ring, Novi Sad J. Math. , Vol. 34, No. 1, 2004, 39-45.