Integrated Intelligent Research (IIR)

International Journal of Computing Algorithm
Volume: 03, Issue: 01 June 2014, Pages: 9-16
ISSN: 2278-2397

A Fuzzy DEMATEL- Trapezoidal Structure for Modeling Cause and Effect Relationships of Youth Violence

A. Felix, A.Victor Devadoss
Department of Mathematics, Loyola College
Email: afelixphd@gmail.com

Abstract - The Decision making trial and evaluation laboratory (DEMATEL) method is a powerful method for capturing the causal relationship between criteria and has been successfully applied to crisp situations. However, in many real life cases, the decision data of human judgments with preferences are often vague so that the traditional ways of using crisp values are inadequate. In this paper, the directed influential degrees between pair wise criteria are expressed in trapezoidal fuzzy numbers is used in DEMATEL method to study on youth violence. Violence is a problem which affects our day today life in a severe way. Even though human beings have been struggling to create civilized societies for many years, they have not able to get free from the influence of violence and aggression yet. Aggressiveness among youth is considered to be a global public health problem in many parts of the world. The existence of violence around a person might provoke him to behave in a more violent way. So in this paper we analyze what makes the youth to be aggressive behavior and involving in violence and its effect

Key words: Causal analysis, Fuzzy theory, Decision making trial and evaluation laboratory DEMATEL, trapezoidal fuzzy number, youth violence.

I. INTRODUCTION

The Battelle Memoria lInstitute conducted the DEMATEL method project throughits Geneva Research Centre (Fontela&Gabus 1976, 1973) [2,4]. TheDEMATEL methodis a potent system analysis tool, which wasaimedatthefragmentedandantagonisticphenomenaofworld societiesandsearchedforintegratedsolutions. It is especially practical and useful for visualizing the structure of complicated causal relationship with matrices or digraphs. In recent years, DEMATEL method has been applied successfully in many fields to analyze correlation among factors and service or requirements in the background software system design (Hori and Schimizu 1999) [10] semiconductor – intellectual property (SIP), mall construction (Li and Tzeng 2009) [14]. On the other hand, The DEMATEL method has also been combined with analytic network process (ANP), goal programming and technique for order preference by similarity to an ideal solution (TOPSIS) to solve problems of core competency analysis (Shieh et.al 2010) [12] andpreference evaluation (Chen et.al 2010; Hsu et.al 2010) [1,6].

When establishinga structuralmodel,human judgmentsfor decidingthe relationship between systems(or sub-systems)are usually given bycrisp values. However,inmanycases, crisp valuesareinadequateinthe real world. Humanjudgmentswithpreferencesareoftenunclearandhardtoestimatebyexactnumericalvalues has createdtheneedforfuzzylogic.Moreover, amoresensibleapproachisto uselinguisticassessmentsinstead of numerical values,inwhichallassessmentsofcriteriainthe problemareevaluated by meansoflinguistic variables (Zadeh,1975) [23]. Emerging research has focused on uncertain linguistic term in group decision making processes such as (Lin, Wu 2004, 2009) [8,9] proposed a Fuzzy extension of the DEMATEL method where uncertain linguistic term converted in to triangular fuzzy number. (Wei et.al 2012)[18] used trapezoidal fuzzy number to develop an extension of a DEMATEL method in an uncertain linguistic environment and using this method we analyzed the cause and effect relationship of youth violence. Therestofthispaperis organizedas follows.InSectiontwo, basic concepts and definitions of linguistic terms and uncertain linguistic terms are introduced. In section three, the classical DEMATEL is presented. In section Four, Fuzzy DEMATEL- trapezoidal method is presented to analyze the correlations among factors in an uncertain linguistic environment. In section five, adaptation of the problem to the model and derived conclusion and scope of study in the final section.

II. PRELIMINARIES

Definition 1

A linguistic variable/term is a variable whose value is not crisp number but word or sentence in a natural language.

Definition 2

If $S=\{s_0, s_1, ..., s_g\}$ be a finite and totally ordered set with odd linguistic terms where s_i denotes the i^{th} linguistic term, $i \in \{0,1,...,g\}$, then we call set S the linguistic term set and $g+1$ the cardinality of S.

It is usually required that set S has the following properties (Herrera et al. 1995; Fan and Liu 2010):

1. The set is ordered: $s_i \,'\geq'\, s_j$, if $i \geq j$, where $'\geq'$ denotes 'greater than or equal to'.
2. There is a negation operator: neg $(s_i) = s_j$ such that j=g-i.
3. Maximization operator: $\max\{s_i, s_j\} = s_i$ if $s_i \,'\geq'\, s_j$.
4. Minimization operator: $\min\{s_i, s_j\} = s_i$ if $s_i \,'\leq'\, s_j$, where $'\leq'$ denotes 'less than or equal to'.

The uncertain linguistic term is a generalization of cognitional expressions to fuzziness and uncertainty. We introduce its definition of uncertain linguistic term below.

International Journal of Computing Algorithm
Volume: 03, Issue: 01 June 2014, Pages: 9-16
ISSN: 2278-2397

Definition 3

Let $\tilde{S}=\{s_l, s_{l+1}, ..., s_u\}$ where $s_l + s_{l+1}, ..., s_u \in S$, $s_l\ '\geq'\ s_u$, s_l and s_u are the lower and upper limits, respectively, $l,u \in \{0,1,...,g\}$. Then we call $\tilde{S}$ the uncertain linguistic term. For simplicity, we express $\tilde{S}$ as $[s_l, s_u]$. Here, the greater $u-l$ is, the greater the fuzziness and uncertainty degree of $[s_l, s_u]$ will be. Particularly, if $l=u$, then $\tilde{S}$ is reduced to a certain linguistic term. For example, in the process of a venture decision , experts may use linguistic term set $S=\{s_0:$ No influence, $s_1:$ Low, $s_2:$ Very High, $s_3:$ High, $s_4:$ Very high} to express his/her opinion on the correlation between cause and effect of youth violence. One expert's judgment may be 'at least High', which can be expressed by an uncertain linguistic term $[s_3, s_4]$. If his/her judgment is 'High', then it can be expressed by an uncertain linguistic term $[s_3, s_3]$. To process the decision information in the form of uncertain linguistic terms, Fan and Liu (2010) provided a theoretical analysis on the union operation of trapezoidal fuzzy numbers. In their research work, an uncertain linguistic term $[s_l, s_u]$ is viewed as a union of several trapezoidal fuzzy numbers. In other word, an uncertain linguistic term $[s_l, s_u]$ can be expressed as a corresponding trapezoidal fuzzy number using the following formula (Fan and Liu 2010): $a_{lu}=(a^1_{lu}, a^2_{lu}, a^3_{lu}, a^4_{lu})$

$$\mu_{\tilde{A}}(x)=\begin{cases} 0, & x<a^1_{lu}\ or\ x>a^4_{lu} \\ \dfrac{x-a^1_{lu}}{a^2_{lu}-a^1_{lu}}, & a^1_{lu}\leq x\leq a^2_{lu} \\ 1 & a^2_{lu}\leq x\leq a^3_{lu} \\ \dfrac{a^4_{lu}-x}{a^4_{lu}-a^3_{lu}} & a^3_{lu}\leq x\leq a^4_{lu} \end{cases} \qquad (1)$$

Where $l,u \in \{0,1,...,g\}$. The derived trapezoidal fuzzy number from the uncertain linguistic term $[s_l, s_u]$ is shown in Fig .1. Thus, the aggregation operations of uncertain linguistic terms can be achieved by the operations of trapezoidal fuzzy numbers.

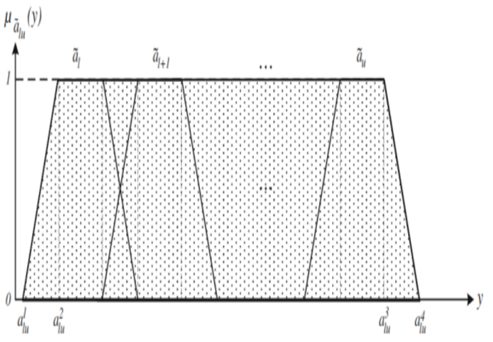

Fig 1 trapezoidal fuzzy number.

Furthermore, in light of the research work by Kaufman and Gupta (1985) [7] and Zadeh (1965) [23], we provide some theorems on the operations of uncertain linguistic terms in the following:

Table 1: Fuzzy Linguistic scale

Linguistic terms	Linguistic values
No influence	(0 , 0, 0, 0, 0.25)
Very low influence	(0, 0, 0.25, 0.5)
low influence	(0, 0.25, 0.5, 0.75)
High influence	(0.25, 0.5, 0.75, 1)
Very high influence	(0.5, 0.75, 1, 1)

Theorem 1

Let $[s_l, s_u]$ and $[s_\alpha, s_\beta]$ be two arbitrary uncertain linguistic terms , and $(a^1_{lu}, a^2_{lu}, a^3_{lu}, a^4_{lu})$ $(a^1_{\alpha\beta}, a^2_{\alpha\beta}, a^3_{\alpha\beta}, a^4_{\alpha\beta})$ be their corresponding trapezoidal fuzzy numbers ; then the addition operations of $[s_l, s_u]$ and $[s_\alpha, s_\beta]$, denoted $[s_l, s_u]\oplus[s_\alpha, s_\beta]$, can yield another trapezoidal fuzzy number, i.e.,

$$[s_l, s_u]\oplus[s_\alpha, s_\beta]=\left(a^1_{lu}+a^1_{\alpha\beta}, a^2_{lu}+a^2_{\alpha\beta}, a^3_{lu}+a^3_{\alpha\beta}, a^4_{lu}+a^4_{\alpha\beta}\right). \qquad (2)$$

Here, notation $'\oplus'$ denotes addition operation of uncertain linguistic terms.

Theorem 2

Let $[s_l, s_u]$ and $[s_\alpha, s_\beta]$ be two arbitrary uncertain linguistic terms, and $(a^1_{lu}, a^2_{lu}, a^3_{lu}, a^4_{lu})$ and $(a^1_{\alpha\beta}, a^2_{\alpha\beta}, a^3_{\alpha\beta}, a^4_{\alpha\beta})$ be their corresponding trapezoidal fuzzy numbers; then the subtraction operations of $[s_l, s_u]$ and $[s_\alpha, s_\beta]$, denoted as $[s_l, s_u]$ $[s_\alpha, s_\beta]$, can yield another trapezoidal fuzzy numbers, i.e.,

$$[s_l, s_u]-[s_\alpha, s_\beta]=(a^1_{lu}-a^4_{\alpha\beta}, a^2_{lu}-a^3_{\alpha\beta}, a^3_{lu}-a^2_{\alpha\beta}, a^4_{lu}-a^1_{\alpha\beta}). \qquad (3)$$

Here,- notation denotes subtraction operation of uncertain linguistic terms.

Theorem 3

Let $[s_l, s_u]$ be an uncertain linguistic term, $(a^1_{lu}, a^2_{lu}, a^3_{lu}, a^4_{lu})$ be its corresponding trapezoidal fuzzy number, and λ be a crisp number $(\lambda>0)$; then $\lambda\otimes[s,s]=(\lambda a^1_{lu}, \lambda a^2_{lu}, \lambda a^3_{lu}, \lambda a^4_{lu})$. (4)

Here, notation $'\otimes'$ denotes multiplication operation of uncertain linguistic terms.

III. THE CLASSICAL DEMATEL METHOD

In this section, the principal and procedure of the classical DEMATEL are presented. Suppose that the set of factors is $F=\{F_1, F_2, ... F_n\}$ and the correlation among factors can be characterized by a graph as a Fig 3. In Fig 3 the arrowed line linked two factors represents that there exists a correlation between them, and the width of the line represents the intensity. Especially, the direction of the arrowed line shows the influence relationship.

International Journal of Computing Algorithm
Volume: 03, Issue: 01 June 2014, Pages: 9-16
ISSN: 2278-2397

The procedure of the classical DEMATEL method is presented as follows (Fontela and Gabus 1976; Gabus and Fontela 1972, 1973) [2,3,4];

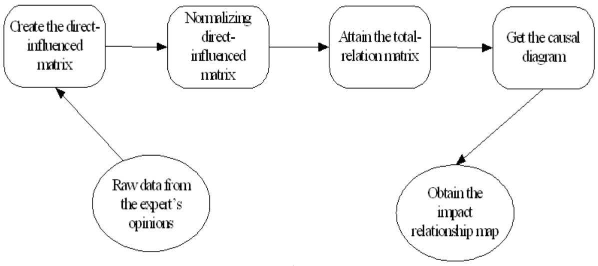

Fig 2 Illustrate steps of DEMATEL in diagram

Step1: set up the initial direct- relation matrix.

Let z_{ij} represent the judgment on the existence and intensity of the correlation between factors F_i and F_j.

Particularly, there does not exist a correlation between F_i and itself. From this, the initial direct-relation matrix $Z=[z_{ij}]_{nxn}$ can be built up.

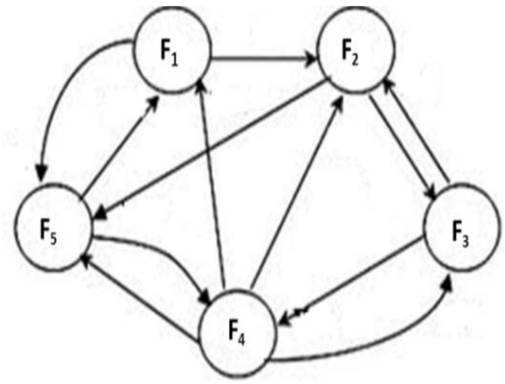

Fig 3 Causal Diagram

Step 2: Construct the normalized direct-relation matrix.

Let $X=[x_{ij}]_{nxn}$ be the normalized direct relation matrix, and x_{ij} is calculate by

$$x_{ij} = z_{ij} / \max_{1 \le i \le n} \left\{ \sum_{j=1}^{n} z_{ij} \right\}, \; i, j = 1, 2, ..., n \tag{5}$$

Where $0 \le x_{ij} \le 1$, generally, the condition of in equation is satisfied in real world particularly, matrix X is characterized as a sub-stochastic matrix obtained from an absorbing Markov chain matrix by deleting all rows and columns associated with absorbing states [2,4]

It satisfies the following properties:

1. $\lim_{n\to\infty} X^n = O$, Where O is the null matrix.
2. $\lim_{n\to\infty}(1 + X^1 + X^2 + ... + X^n) = (1 - X)^{-1}$, Where I is the identity matrix.

Step 3: construct the overall-relation matrix.

Let $T=[t_{ij}]_{nxn}$ be the overall-relation matrix, and it can be derived by

$$T = \lim_{n\to\infty}(1 + X^1 + X^2 + ... + X^n) = X(1 - X)^{-1}, \tag{6}$$

Where t_{ij} denotes the overall intensity of correlation between factors F_i and F_j.

Let c_i denote the overall intensity of correlation between F_i and F_j influences others and it can be calculated by

$$c_i = \sum_{j=1}^{n} t_{ij}, \quad i = 1, 2...n \tag{7}$$

Let h_i denote the overall intensity that factor F_i is influenced by others, and it can be derived by

$$h_i = \sum_{j=1}^{n} t_{ji}, \quad i = 1, 2...n \tag{8}$$

Step 4: Calculate the prominence and relation of each factor.

Let p_i be the prominence of factor F_i, and it is calculated by

$$p_i = c_i + h_i, \; 1 = 1, 2, ...n \tag{9}$$

Based on prominence P_i, the importance of factor F_i is determined. The large P_i, the more important factor *Fi*. If the importance of a factor is greater, then the decision maker should pay much attention to it.

Let r_i be the relation of factor F_i and it can be obtained by

$$r_i = c_i - h_i, \; 1 = 1, 2, ...n \tag{10}$$

Relation r_i is an indicator that is used to judge role of factor F_i, if $r_i > 0$, then F_i is a cause factor. If if$r_i < 0$, then F_i is an effect factor.

Step 5: Construct the causal diagram

Based on prominence p_i and relation r_i, a causal diagram can be plotted to visualize the importance and classification of all factor. In the causal diagram, horizontal axis P denote the importance of factors the importance of factors while vertical axis R denotes the sort of factors.

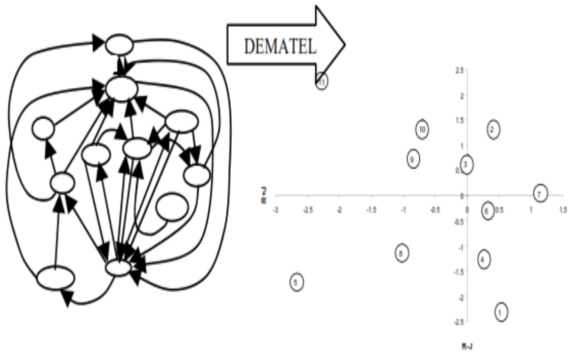

Fig 4 Structure of the DEMATEL method

IV. FUZZY DEMATEL- TRAPEZOIDAL METHOD

In this section, an extended DEMATEL method is given to analyze the correlations among factors in an uncertain linguistic environment.

Step 1: Set up the initial uncertain direct-relation matrix $\hat{Z}_k = [\hat{z}_{kij}]_{n\times n}$

Let $F = \{F_1, F_2, ..., F_n\}$ be a finite set of factors, where F_i denotes the ith factor, $i \in \{1, 2, .., n\}$; $E = \{E_1, E_2, ..., E_m\}$ be a finite set of experts, where E_k denotes the k[th] expert, $k \in \{1, 2, ..., m\}$; $S = \{s_0, s_1, ..., s_g\}$ be a pre-established set of linguistic terms, where s_l denotes the *l*th linguistic term , $l \in \{0, 1, ..., g\}$. We assume that the experts have the identical importance and use set S to express their judgments on the intensities of correlations among factors. Let $\hat{z}_{kij}$ represent the judgment on the intensity of the correlation between factors F_i and F_j provided by expert E_k, k=1,2,…,m, i,j=1,2,…n.

International Journal of Computing Algorithm
Volume: 03, Issue: 01 June 2014, Pages: 9-16
ISSN: 2278-2397

If there does not exist a correlation between F_i and F_j, then we znote $\hat{z}_{kij}$='–'. Particularly, let $\hat{z}_{kij}$='–'. represent that there does not exist a correlation between F_i and itself. From this, the initial $='-'$ uncertain direct –relation matrix $\hat{Z}_k = [\hat{z}_{kij}]_{n\times n}$ Provided by expert E_k can be set up, i.e.,

$$\hat{Z}_k = [\hat{z}_{kij}]_{n\times n} = \begin{matrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{matrix} \overset{\begin{matrix} F_1 & F_2 & \dots & F_n \end{matrix}}{\begin{bmatrix} - & \hat{z}_{k12} & \cdots & \hat{z}_{k1n} \\ \hat{z}_{k21} & - & \cdots & \hat{z}_{k2n} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{z}_{kn1} & \hat{z}_{kn2} & \cdots & - \end{bmatrix}}, \ k=1,2,\dots,m$$

Step 2: Transform matrix into matrix $\hat{Z}_k = [\hat{z}_{kij}]_{n\times n}$ into $\tilde{Z}_k = [\tilde{z}_{kij}]_{n\times n}$

By Eq. 1, initial uncertain direct-relation matrices are transformed into the form of trapezoidal fuzzy numbers, i.e., $\hat{Z}_k = [\hat{z}_{kij}]_{n\times n}$ is transformed into $\hat{Z}_k = [\hat{z}_{kij}]_{n\times n}$, $\hat{z}_{kij} = (z^1_{kij}, z^2_{kij}, z^3_{kij}, z^4_{kij})$ k=1,2,….,m i, j=1,2,…,n. Particularly, $\hat{z}_{kij} = '-'$ is transformed into $\hat{z}_{kij} = (0,0,0,0)$. By Eq. 2 and 4, transformed individual uncertain direct-relation matrices $\hat{Z}_1, \hat{Z}_2, ..., \hat{Z}_m$ are aggregated into a group uncertain direct-relation matrix $\hat{Z}_k = [\hat{z}_{kij}]_{n\times n}$.

Step 3: Construct the group uncertain direct relation matrix $\tilde{Z}_k = [\tilde{z}_{kij}]_{n\times n}$

By Eq. 2 and 4, transformed individual uncertain direct-relation matrices $\tilde{Z}_1, \tilde{Z}_2, ..., \tilde{Z}_m$ are aggregated into a group uncertain direct-relation matrix $\tilde{Z}_k = [\tilde{z}_{kij}]_{n\times n}$. If we note that $\tilde{z}_k = (z^1_{kij}, z^2_{kij}, z^3_{kij}, z^4_{kij})$ then $z^1_{kij}, z^2_{kij}, z^3_{kij}$ and z^4_{kij} are calculated by

$$z^1_{ij} = \frac{1}{m}\sum_{k=1}^{m} z^1_{kij}, \ i,j=1,2\dots,n, \quad , \tag{11a}$$

$$z^2_{ij} = \frac{1}{m}\sum_{k=1}^{m} z^2_{kij}, \quad i,j=1,2,\dots,n, \tag{11b}$$

$$z^3_{ij} = \frac{1}{m}\sum_{k=1}^{m} z^3_{kij}, \ i,j=1,2,\dots,n \tag{11c}$$

$$z^4_{ij} = \frac{1}{m}\sum_{k=1}^{m} z^4_{kij}, \ i,j=1,2,...,n \tag{11d}$$

Step 4: construct normalized uncertain overall-relation matrix

By Eq.4, group uncertain direct-relation matrix $\tilde{Z}_k = [\tilde{z}_{kij}]_{n\times n}$ is changed into the normalized uncertain direct-relation matrix $\tilde{X} = [\tilde{x}_{ij}]_{n\times n}$. If we note that $\tilde{x}_{ij} = (x^1_{ij}, x^2_{ij}, x^3_{ij}, x^4_{ij})$ then $x^1_{ij}, x^2_{ij}, x^3_{ij}$ and x^4_{ij} are expressed by

$$x^1_{ij} = z^1_{ij} \Big/ \max_{1\le i\le n}\left\{\sum_{j=1}^{n} z^8_{ij}\right\}, \ i,j=1,2,\dots,n, \tag{12a}$$

$$x^2_{ij} = z^2_{ij} \Big/ \max_{1\le i\le n}\left\{\sum_{j=1}^{n} z^8_{ij}\right\}, \ i,j=1,2,\dots,n, \tag{12b}$$

$$x^3_{ij} = z^3_{ij} \Big/ \max_{1\le i\le n}\left\{\sum_{j=1}^{n} z^8_{ij}\right\}, \ i,j=1,2,\dots,n, \tag{12c}$$

$$x^4_{ij} = z^4_{ij} \Big/ \max_{1\le i\le n}\left\{\sum_{j=1}^{n} z^8_{ij}\right\}, \ i,j=1,2,\dots,n \tag{12d}$$

Where $\max_{1\le i\le n}\left\{\sum_{j=1}^{n} z^4_{ij}\right\} \neq 0$ and $0 \le x^1_{ij} \le x^2_{ij} \le x^3_{ij} \le x^4_{ij} < 1$.

We decompose matrix $\tilde{X}$ into four crisp value matrices X^1, X^2, X^3 and X^4, i.e.

$$X^1 = \begin{bmatrix} 0 & x^1_{12} & \cdots & x^1_{1n} \\ x^1_{21} & 0 & \cdots & x^1_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x^1_{n1} & x^1_{n2} & \cdots & 0 \end{bmatrix}, \quad X^2 = \begin{bmatrix} 0 & x^2_{12} & \cdots & x^2_{1n} \\ x^2_{21} & 0 & \cdots & x^2_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x^2_{n1} & x^2_{n2} & \cdots & 0 \end{bmatrix}$$

$$X^3 = \begin{bmatrix} 0 & x^3_{12} & \cdots & x^3_{1n} \\ x^3_{21} & 0 & \cdots & x^3_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x^3_{n1} & x^3_{n2} & \cdots & 0 \end{bmatrix} \quad X^4 = \begin{bmatrix} 0 & x^4_{12} & \cdots & x^4_{1n} \\ x^4_{21} & 0 & \cdots & x^4_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x^4_{n1} & x^4_{n2} & \cdots & 0 \end{bmatrix}$$

In order to achieve the computation of $\tilde{X}^{\tau}$ using the multiplication operation of crisp value matrices

According to the classical DEMATEL method, the uncertain overall-relation matrix $\tilde{T}$ is defined as

$$\tilde{T} = \lim_{\tau\to\infty}\left((\tilde{X})^1 + (\tilde{X})^2 + ... + (\tilde{X})^{\tau}\right) \tag{13}$$

Step 5: construct uncertain overall relation matrix 14a-d $\tilde{T} = [\tilde{t}_{ij}]_{n\times n}$

Let $\tilde{T} = \begin{bmatrix} \tilde{t}_{11} & \tilde{t}_{12} & \cdots & \tilde{t}_{1n} \\ \tilde{t}_{21} & \tilde{t}_{22} & \cdots & \tilde{t}_{21} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{t}_{n1} & \tilde{t}_{n1} & \cdots & \tilde{t}_{nn} \end{bmatrix}$, where $\tilde{t}_{ij} = \left(t^1_{ij}, t^2_{ij}, t^3_{ij}, t^4_{ij},\right)$, Then

$$\left[t^1_{ij}\right]_{n\times n} = X^1\left(I - X^1\right)^{-1}, i, j = 1, 2, ..., n, \tag{14a}$$

$$\left[t^2_{ij}\right]_{n\times n} = X^2\left(I - X^2\right)^{-1}, i, j = 1, 2, ..., n, \tag{14b}$$

$$\left[t^3_{ij}\right]_{n\times n} = X^3\left(I - X^3\right)^{-1}, i, j = 1, 2, ..., n, \tag{14c}$$

$$\left[t^4_{ij}\right]_{n\times n} = X^4\left(I - X^4\right)^{-1}, i, j = 1, 2, ..., n, \tag{14d}$$

Step 6: determine the overall intensities of influencing and influenced correlation of factors F_i, $\tilde{c}_i$ and $\tilde{r}_i$, $i = 1, 2, ..., n$,

International Journal of Computing Algorithm
Volume: 03, Issue: 01 June 2014, Pages: 9-16
ISSN: 2278-2397

Let $\tilde{c}_i$ represent the overall intensity that factor F_i influence others, and then it can be determined by calculating the sum of each row of matrix $\tilde{T}$. If we denote $\tilde{c}_i = \left(c_i^1, c_i^2, c_i^3, c_i^4\right)$, then c_i^1, c_i^2, c_i^3 and c_i^4 are calculated by $c_i^1 = \sum_{j=1}^{n} t_{ij}^1, i = 1, 2, ..., n,$ $c_i^2 = \sum_{j=1}^{n} t_{ij}^2, i = 1, 2, ..., n,$ $c_i^3 = \sum_{j=1}^{n} t_{ij}^3, i = 1, 2, ..., n,$ $c_i^4 = \sum_{j=1}^{n} t_{ij}^4, i = 1, 2, ..., n,$

(15 a- d)

Let $\tilde{h}_j$ represent the overall intensity that factor F_j is influenced by others, and it can be determined by calculating the sum of each column of matrix $\tilde{T}$. If we express $\tilde{h}_j = \left(h_j^1, h_j^2, h_j^3, h_j^4\right)$, then h_j^1, h_j^2, h_j^3 and h_j^4 are calculated by $h_j^1 = \sum_{j=1}^{n} t_{ij}^1, j = 1, 2, ..., n,$ $h_j^2 = \sum_{j=1}^{n} t_{ij}^2, j = 1, 2, ..., n,$ $h_j^3 = \sum_{j=1}^{n} t_{ij}^3, j = 1, 2, ..., n,$ $h_j^4 = \sum_{j=1}^{n} t_{ij}^4, j = 1, 2, ..., n,$ (16a- d)

Step7: determine the uncertain prominence and relation of each factor $\tilde{p}_i$ and $\tilde{c}_i$

Furthermore, let $\tilde{p}_i$ be the uncertain prominence of factor F_i; then it can be determined by calculating the sum of $\tilde{c}_i$ and $\tilde{h}_i$. If we note $\tilde{p}_i = \left(p_i^1, p_i^2, p_i^3, p_i^4\right)$, then p_i^1, p_i^2, p_i^3 and p_i^4 are represented by

$$p_i^1 = c_i^1 + h_i^1,\ i = 1,2,...,n,$$
$$p_i^2 = c_i^2 + h_i^2,\ i = 1,2,...,n$$
$$p_i^3 = c_i^3 + h_i^3,\ i = 1,2,...,n,$$
$$p_i^4 = c_i^4 + h_i^4,\ i = 1,2,...,n \quad \text{(17 a-d)}$$

Let $\tilde{r}_i$ be the uncertain relation of factor F_i; then it can be determined by calculating the difference between $\tilde{c}_i$ and $\tilde{h}_i$. If we note that $\tilde{r}_i = \left(r_i^1, r_i^2, r_i^3, r_i^4\right)$, then r_i^1, r_i^2, r_i^3 and r_i^4 are calculated by

$$r_i^1 = c_i^1 - h_i^1,\ i = 1,2,...,n,\ r_i^2 = c_i^2 - h_i^2,\ i = 1,2,...,n,\ r_i^3 = c_i^3 - h_i^3,\ i = 1,2,...,n,$$
$$r_i^4 = c_i^4 - h_i^4,\ i = 1,2,...,n,\ \text{(18a-d)}$$

Step 8: determine the crisp prominence and relation each factor p_i and r_i

To obtain the importance ranking order and the classification of factor, we usually convert the final fuzzy data into crisp values. Using the centroid (center of gravity) methods (Yager and Filev 1994) for defuzzifyingTrepizoidal fuzzy numbers, the crisp values of prominence and relation of factor F_i, denoted by p_i and r_i are obtained by

$$p_i = \frac{1}{4}\left(p_i^1 + p_i^2 + p_i^3 + p_i^4\right),\ i = 1,2,...,n.$$
$$r_i = \frac{1}{4}\left(r_i^1 + r_i^2 + r_i^3 + r_i^4\right),\quad i = 1,2,...,n \quad \text{(19-20)}$$

Step 9: construct the causal diagram based on p_i and r_i

Using the p_i and r_i the importance and classification of factors are, respectively, determined and a causal diagram can be derived for the visualization of the importance and classification of factors.

V. ADAPTATION OF THE PROBLEM TO THE MODEL

We have interviewed and collected a data from 100 youth in and around Chennai to analyses what is the causes and effect of youth violence. What they spell out that we have chosen as the attributes.

C_1 Involvement of homicide / murdering, /C_2 Poor monitoring and supervision of children by parents. / C_3Academic failure / dropping out of school, / C_4 Delinquent peers / Gang membership, / C_5 Ill treatment / no motivation by teacher, / C_6Poverty / unemployment, C_7Opportunities are denied / unfulfilled curiosity, / C_8Seeking recognition, C_9 Audit for drugs and alcohol, / C_{10} Castisem / inequality, / C_{11} Political or religious association, / C_{12} Involvement in other forms of antisocial behaviour such as terrorism, robbery etc…, / C_{13} Poor behaviour control, / C_{14}Depression, / C_{15}Aggressive behaviour, and C_{16} Parental substance or criminality

The major nine steps were conducted as following.

Then, the experts called to give their judgements on the existences and intensities of the correlation among the risk factors and the opinion provided by the experts are collected by the questionnaires. The experts marked one are several adjacent linguistic terms of a pre-determined linguistic term set to express their judgements on the strength of correlation between any two risk factors, Here, S = { s_0: No Influence, s_1: Very Low, s_2: Low, s_3: High, s_1: Very High}. For example, one of the experts revealed that when parents are failed to monitor and supervise of their children, children's will get the relationship with the bad gang membership with the High Intensities. Nowadays, most of the youths are involving robbery, terrorism… etc., because of poverty and the job opportunities are denied with very high intensities.

International Journal of Computing Algorithm
Volume: 03, Issue: 01 June 2014, Pages: 9-16
ISSN: 2278-2397

Step 1: Set up initial uncertain direct-relation matrix

Table2: Initial uncertain direct-relation matrix provided by expert

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}
C_1	-	VH	H	VH	VL	L	L	H	VH	L	VH	VL	VH	VH	H	VH
C_2	VL	-	H	H	L	VL	VL	VL	VH	L	VH	H	H	VL	H	H
C_3	H	H	-	VH	VH	H	L	L	H	L	H	VH	H	VL	VL	H
C_4	VL	H	VH	-	H	L	VL	VH	VH	H	H	VH	VH	L	VH	VH
C_5	L	L	VH	VH	-	L	H	L	H	H	VL	L	L	H	L	VL
C_6	VL	L	H	H	L	-	L	L	H	VL	H	VH	VL	H	L	H
C_7	VL	H	L	H	L	H	-	L	VL	H	L	VH	VL	H	H	VL
C_8	H	VL	VL	VH	L	VL	L	-	VL	H	VH	H	H	L	H	VL
C_9	H	VH	H	VH	H	H	VH	L	-	VL	H	H	VH	VL	H	VH
C_{10}	VH	L	VL	VH	VL	VL	VH	H	L	-	VH	H	VH	VL	L	VH
C_{11}	L	L	L	VH	L	VL	L	H	H	VH	-	VH	VL	L	L	VH
C_{12}	L	H	VH	VH	H	VH	H	L	VL	L	VL	-	L	L	VH	VH
C_{13}	VL	H	L	H	H	VL	L	VL	H	H	L	L	-	VL	H	H
C_{14}	H	VL	VL	H	VH	VL	VL	L	VH	L	L	L	H	-	H	VH
C_{15}	H	VL	VL	H	VH	VL	VL	L	L	H	H	VL	H	L	-	VH
C_{16}	H	VH	H	VH	H	H	L	VL	VL	VH	H	VH	H	VL	VH	-

Step:2 and 3 transform the uncertain direct – relation matrix into fuzzy Trapezoidal number.

Table 3: transform the uncertain direct-relation matrix $\tilde{Z}_1$

	C_1	C_2	C_3	...	C_{16}
C_1	-	(0.5,0.75,1,1)	(0.25,0.5,0.75,1)	…	(0.5,0.75,1,1)
C_2	(0,0.25,0.5,0.75)	-	(0.25,0.5,0.75,1)	…	(0.25,0.5,0.75,1)
C_3	(0.25,0.5,0.75,1)	(0.25,0.5,0.75,1)	-	…	(0.25,0.5,0.75,1)
C_4	(0,0.25,0.5,0.75)	(0.25,0.5,0.75,1)	(0.5,0.75,1,1)	…	(0.5,0.75,1,1)
C_5	(0,0,0,0.25)	(0,0,0,0.25)	(0.5,0.75,1,1)	…	(0,0.25,0.5,0.75)
C_6	(0,0.25,0.5,0.75)	(0,0,0,0.25)	(0.25,0.5,0.75,1)	…	(0.25,0.5,0.75,1)
C_7	(0,0.25,0.5,0.75)	(0.25,0.5,0.75,1)	(0,0,0.25,0.5)	…	(0,0.25,0.5,0.75)
C_8	(0.25,0.5,0.75,1)	(0,0.25,0.5,0.75)	(0,0.25,0.5,0.75)	…	(0,0.25,0.5,0.75)
C_9	(0.25,0.5,0.75,1)	(0.5,0.75,1,1)	(0.25,0.5,0.75,1)	…	(0.5,0.75,1,1)
C_{10}	(0.5,0.75,1,1)	(0,0,0,0.25)	(0,0.25,0.5,0.75)	…	(0.5,0.75,1,1)
C_{11}	(0,0,0,0.25)	(0,0,0.25,0.5)	(0,0,0,0.25)	…	(0.5,0.75,1,1)
C_{12}	(0,0,0,0.25)	(0.25,0.5,0.75,1)	(0.5,0.75,1,1)	…	(0.5,0.75,1,1)
C_{13}	(0,0.25,0.5,0.75)	(0.25,0.5,0.75,1)	(0,0,0.25,0.5)	…	(0.25,0.5,0.75,1)
C_{14}	(0.25,0.5,0.75,1)	(0,0.25,0.5,0.75)	(0,0.25,0.5,0.75)	…	(0.5,0.75,1,1)
C_{15}	(0.25,0.5,0.75,1)	(0,0.25,0.5,0.75)	(0,0.25,0.5,0.75)	…	(0.5,0.75,1,1)
C_{16}	(0.25,0.5,0.75,1)	(0.5,0.75,1,1)	(0.25,0.5,0.75,1)	…	-

Step 4: construct normalized uncertain direct-relation matrix. We change $\tilde{Z}$ in to normalized direct- relation matrix $\tilde{X}$ using the Equs (12a-h)

Table 4: Normalized uncertain direct-relation matrix $\tilde{X}$

	C_1	C_2	C_3	...	C_{16}
C_1	(0, 0, 0, 0)	(0.033, 0.05, 0.0667, 0.0667)	(0.02, 0.033, 0.05, 0.067)	...	(0.033, 0.05, 0.067, 0.067)
C_2	(0, 0.017, 0.033, 0.05)	(0, 0, 0, 0)	(0.02, 0.033,0.05,0.067	...	(0.017, 0.033, 0.05, 0.067)
C_3	(0.017, 0.033, 0.05, 0.067)	(0.017, 0.033, 0.05, 0.067)	(0, 0, 0, 0)	...	(0.017, 0.033,0.05, 0.067)
C_4	(0, 0.017, 0.033, 0.05)	(0.017, 0.033, 0.05, 0.067)	(0.03,0.05,0.067,0.067)	...	(0.033, 0.05, 0.067, 0.067)
C_5	(0, 0, 0, 0.017)	(0, 0, 0, 0.017)	(0.03, 0.05,0.067,0.067)	...	(0, 0.017, 0.03, 0.05)
C_6	(0, 0.017, 0.033, 0.05)	(0, 0, 0, 0.017)	(0.02, 0.033,0.05, 0.067)	...	(0.017, 0.03, 0.05, 0.07)
C_7	(0, 0.017, 0.033, 0.05)	(0.017, 0.033, 0.05, 0.067)	(0, 0, 0.017, 0.033)	...	(0, 0.017, 0.03, 0.05)
C_8	(0.0167, 0.033, 0.05, 0.067)	(0, 0.017, 0.033, 0.05)	(0, 0.017, 0.033, 0.05)	...	(0, 0.017, 0.03, 0.05)
C_9	(0.0167, 0.033, 0.05, 0.067)	(0.033, 0.05, 0.0667, 0.0667)	(0.02, 0.033, 0.05, 0.067)	...	(0.03, 0.05, 0.07, 0.07)
C_{10}	(0.033, 0.05, 0.067, 0.067)	(0, 0, 0, 0.017)	(0, 0.017, 0.033, 0.05)	...	(0.03, 0.05, 0.07, 0.07)
C_{11}	(0, 0, 0, 0.0167)	(0, 0, 0.0167, 0.0333)	(0, 0, 0, 0.017)	...	(0.03, 0.05, 0.07, 0.07)
C_{12}	(0, 0, 0, 0.0167)	(0.017, 0.033, 0.05, 0.067)	(0.03, 0.05, 0.067, 0.067)	...	(0.03, 0.05, 0.07, 0.07)
C_{13}	(0, 0.017, 0.033, 0.05)	(0.017, 0.033, 0.05, 0.067)	(0, 0, 0.017, 0.033)	...	(0.017, 0.033,0.05, 0.067)
C_{14}	(0.0167, 0.033, 0.05, 0.067)	(0, 0.017, 0.033, 0.05)	(0, 0.017, 0.033, 0.05)	...	(0.03, 0.05, 0.07, 0.07)
C_{15}	(0.0167, 0.033, 0.05, 0.067)	(0, 0.017, 0.033, 0.05)	(0, 0.017, 0.033, 0.05)	...	(0.03, 0.05, 0.07, 0.07)
C_{16}	(0.0167, 0.033, 0.05, 0.067)	(0.033, 0.05, 0.0667, 0.0667)	(0.02, 0.033, 0.05, 0.067)	...	(0, 0, 0, 0)

Step 5: construct uncertain overall-relation matrix.Using Equs (14a-h), uncertain overall-relation matrix $\tilde{T}$ is constructed.

International Journal of Computing Algorithm
Volume: 03, Issue: 01 June 2014, Pages: 9-16
ISSN: 2278-2397

Table 5: uncertain overall – relation $\tilde{T}$ matrix

	C_1	C_2	C_3	...	C_{16}
C_1	(0.003, 0.017, 0.061, 0.199)	(0.04, ,0.07, 0.134, 0.27)	(0.02, 0.053, 0.12, 0.28)	...	(0.041, 0.08, 0.16, 0.31)
C_2	(0.002, 0.03, 0.085, 0.246)	(0.003, 0.02, 0.06, 0.2)	(0.019, 0.05, 0.1, 0.28)	...	(0.022, 0.06, 0.13, 0.31)
C_3	(0.018, 0.045, 0.1, 0.26)	(0.02, 0.05, 0.12, 0.264)	(0.005, 0.02, 0.068, 0.214)	...	(0.023, 0.06, 0.135, 0.31)
C_4	(0.004, 0.035, 0.096, 0.253)	(0.02, 0.05, 0.12, 0.27)	(0.037, 0.07, 0.14, 0.28)	...	(0.041, 0.08, 0.163, 0.32)
C_5	(0.002, 0.01, 0.04, 0.183)	(0.002, 0.012, 0.05, 0.19)	(0.035, 0.06, 0.11, 0.24)	...	(0.004, 0.034, 0.1, 0.25)
C_6	(0.001, 0.027, 0.08,0.224)	(0.003, 0.014, 0.052, 0.2)	(0.02, 0.05, 0.1, 0.252)	...	(0.021, 0.053, 0.12, 0.28)
C_7	(0.001, 0.028, 0.08, 0.23)	(0.02, 0.05, 0.1, 0.25)	(0.003, 0.015, 0.074, 0.23)	...	(0.005, 0.04, 0.11, 0.28)
C_8	(0.018, 0.045, 0.01, 0.25)	(0.002, 0.03, 0.09, 0.24)	(0.002, 0.03, 0.09, 0.25)	...	(0.006, 0.04, 0.114, 0.285)
C_9	(0.018, 0.049, 0.111, 0.28)	(0.04, 0.07, 0.14, 0.28)	(0.022, 0.054, 0.125, 0.3)	...	(0.04, 0.08, 0.164, 0.33)
C_{10}	(0.04, 0.063, 0.12, 0.25)	(0.005, 0.02, 0.06, 0.210)	(0.004, 0.034, 0.1, 0.25)	...	(0.04, 0.075, 0.15, 0.3)
C_{11}	(0.003, 0.012, 0.04, 0.175)	(0.003, 0.014, 0.06, 0.2)	(0.004, 0.015, 0.052, 0.19)	...	(0.04, 0.07, 0.13, 0.26)
C_{12}	(0.002, 0.013, 0.05, 0.19)	(0.02, 0.05, 0.1, 0.24)	(0.037, 0.07, 0.124, 0.25)	...	(0.038, 0.07, 0.14, 0.28)
C_{13}	(0.002, 0.03, 0.081, 0.24)	(0.02, 0.045, 0.1, 0.25)	(0.002, 0.015, 0.08, 0.24)	...	(0.02, 0.052, 0.12, 0.3)
C_{14}	(0.018, 0.045, 0.01, 0.245)	(0.004, 0.03, 0.09, 0.23)	(0.004, 0.033, 0.1, 0.244)	...	(0.037, 0.07, 0.14, 0.29)
C_{15}	(0.018, 0.04, 0.1, 0.25)	(0.003, 0.03, 0.09, 0.24)	(0.003, 0.032, 0.1, 0.25)	...	(0.037, 0.07, 0.142, 0.23)
C_{16}	(0.02, 0.050, 0.112, 0.28	(0.04, 0.07, 0.13, 0.28)	(0.021, 0.055, 0.13, 0.3)	...	(0.008, 0.032, 0.1, 0.27)

Step 6: construct uncertain overall intensities of the influencing and influenced correlations.

Based on $\tilde{T}$, the overall intensities of correlations of factors and form other factors, i.e. $\tilde{D}_i$ and $\tilde{R}_i$ are calculated by Equs (15a-16h) is shown in the table- 6

Step 7: Determine the uncertain prominence and relation of each other Based on $\tilde{D}_i$ and $\tilde{R}_i$ the uncertain prominence and relation, $\tilde{p}_i$ and $\tilde{r}_i$, are calculate by Eqs (17a- 18h) is shown in the table -6.

Step 8: Determine the crisp prominence and relation of each other

Using the Equ 19 -20, the crisp values of prominence and relation, p_i and r_i, are obtained. It is shown in the table-Table.

Table 6: Computational results

$\tilde{D}_i$	$\tilde{R}_i$	$\tilde{p}_i$	$\tilde{r}_i$	p_i	r_i
(0.356, 0.801, 1.795, 4.110)	(0.164, 0.541,1.349,3.759)	(0.520, 1.342,3.144, 7.869)	(-0.192, -0.259, -0.446, -0.351)	3.219	-0.312
(0.212, 0.637, 1.579, 4.105)	(0.237, 0.621, 1.487, 3.815)	(0.449, 1.258, 3.066, 7.920)	(0.024, -0.016, -0.092, -0.289)	3.173	0.093
(0.274, 0.687, 1.616, 4.090)	(0.239, 0.649, 1.602, 4.034)	(0.513, 1.336, 3.218, 8.124)	(-0.035, -0.038, -0.014, -0.056)	3.298	-0.036
(0.374, 0.848, 1.873, 4.256)	(0.495, 1.054, 2.256, 4.850)	(0.869, 1.902, 4.128, 9.107)	(0.122, 0.205, 0.383, 0.594)	4.001	0.326
(0.172, 0.455, 1.219, 3.397)	(0.234, 0.596, 1.460, 3.781)	(0.406, 1.051, 2.679, 7.178)	(0.062, 0.141, 0.241, 0.384)	2.828	0.207
(0.172, 0.509, 1.303, 3.637)	(0.129, 0.497, 1.319, 3.684)	(0.301, 1.005, 2.622, 7.320)	(-0.043, -0.012, 0.016, 0.047)	2.812	0.002
(0.167, 0.527, 1.376, 3.796)	(0.126, 0.389, 1.124, 3.232)	(0.292, 0.915, 2.500, 7.028)	(-0.041, -0.138, -0.252, -0.564)	2.684	-0.249
(0.191, 0.584, 1.502, 3.944)	(0.111, 0.352, 0.906, 2.871)	(0.302, 0.936, 2.409, 6.815)	(-0.080, -0.232, -0.596, -1.072)	2.615	-0.495
(0.352, 0.844, 1.899, 4.468)	(0.270, 0.712, 1.694, 4.168)	(0.622, 1.556, 3.593, 8.636)	(-0.083, -0.132, -0.205, -0.299)	3.602	-0.180
(0.294, 0.712, 1.646, 3.906)	(0.216, 0.564, 1.433, 3.793)	(0.510, 1.276, 3.079, 7.699)	(-0.078, -0.148, -0.213, -0.113)	3.141	-0.138
(0.218, 0.515, 1.236, 3.257)	(0.288, 0.712, 1.621, 4.048)	(0.506, 1.227, 2.856, 7.305)	(0.070, 0.197, 0.385, 0.791)	2.973	0.361
(0.271, 0.632, 1.452, 3.622)	(0.337, 0.775, 1.800, 4.179)	(0.608, 1.407, 3.252, 7.801)	(0.067, 0.143, 0.347, 0.557)	3.267	0.278
(0.149, 0.504, 1.389, 3.894)	(0.295, 0.742, 1.738, 4.240)	(0.443,1.246, 3.126, 8.134)	(0.146, 0.238, 0.349, 0.346)	3.237	0.270
(0.213, 0.586, 1.435, 3.743)	(0.097, 0.397, 1.114, 3.335)	(0.310, 0.984, 2.549, 7.077)	(-0.116, -0.189, -0.321, -0.408)	2.730	-0.258
(0.190, 0.576, 1.488, 3.921	(0.280, 0.675, 1.683, 4.169)	(0.470, 1.252, 3.171, 8.089)	(0.090, 0.099, 0.195, 0.248)	3.245	0.158
(0.334, 0.820, 1.898, 4.464)	(0.420, 0.961, 2.120, 4.649)	(0.754, 1.780, 4.018, 9.113)	(0.086, 0.141, 0.222, 0.185)	3.916	0.159

Step 9: Construct the causal diagram Using the crisp values of prominence and relation, p_i and r_i, drawn the casual diagram is shown

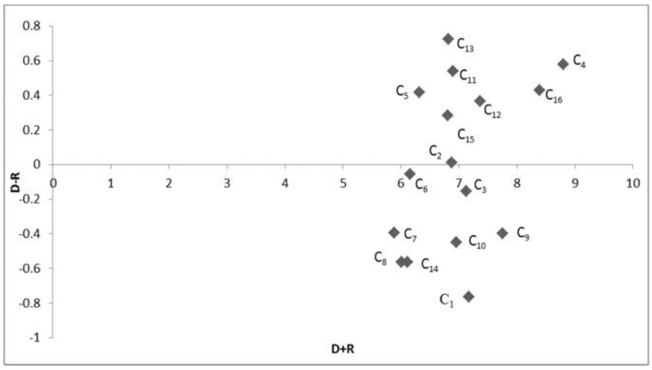

Fig-5 Causal Diagram

VI. RESULTS AND DISCUSSION

The graphical representation (the prominence-causal diagram) and digraphical relationships are now constructed. This step will allow a clearer visualization of the structure and relationships amongst the attributes of youth violence. The evaluation criteria were visually divided into the cause group, including C_2 C_4 C_5 C_{11} C_{12} C_{13} C_{15} C_{16} and the effect group, including C_1 C_3 C_6 C_7 C_8 C_9 C_{10} andC_{14}. From the causal diagram, valuable clues are obtained for making profound decisions so that we can prevent youth violence and give a remedy to them. For example poor monitoring and supervision of the children by parents (C_2) and parental substance and criminality (C_{16}) are the main causes of youths are involving in violence.

International Journal of Computing Algorithm
Volume: 03, Issue: 01 June 2014, Pages: 9-16
ISSN: 2278-2397

V. CONCLUSION

This paper presents an extended DEMATEL method for analysing relationship among factors in an uncertain linguistic environment. With this method, the complex interaction between criteria can be transformed into a visible structural model, making it easier to capture the complexity of a problem, whereby profound decision can be made by the decision maker. Further research may represent linguistic variables by Octagonal fuzzy numbers of membership function and type-2 fuzzy number.

ACKNOWLEDGEMENT

This research is supported by UGC scheme MANF. Award letter F1-17.1/2012-13/MANF-2012-13-CHR-TAM-11197 / (SA-III/Website)

REFERENCE

[1] Chen JK, Chen IS (2010) "Using a novel conjunctive MCDM approach based on DEMATEL, fuzzy ANP, and TOPSIS as an innovation support system for Taiwanese higher education". Expert SystAppl 37(3):1981–1990
[2] Fontela E, Gabus A (1976) "The DEMATEL Observer, DEMATEL 1976 Report". Battelle Geneva Research Centre, Geneva
[3] Gabus A, Fontela E (1972) "World problems an invitation to further thought within the framework of DEMATEL". Battelle Geneva Research Centre, Geneva
[4] Gabus A, Fontela E (1973) "Perceptions of the world problematique: communication procedure, communicating with those bearing collective responsibility (DEMATEL Report no.1)". Battelle Geneva Research Centre, Geneva
[5] Goodman R(1988) " Introduction to stochastic models" Benjamin/ Cummings publishing company, Monolo Park
[6] Hsu YL, Li WC, Chen KW (2010) "Structuring critical success factors of airline safety management system using a hybrid model". Transp Res E LogistTransp Rev 46(2):222–235
[7] Kaufman A, Gupta MM (1985) "Introduction to fuzzy arithmetic: theory and applications". Van Nostrand Reinhold, New York
[8] Lin, Wu (2004) "A fuzzy Extension of the DEMATEL Method for Group Decision – Making"
[9] Lin, Wu (2008) "A causal analytical method for decision making under fuzzy environment". Expert system with Applic.205-213
[10] Hori S, Shimizu Y (1999) "Designing methods of human interface for supervisory control systems". Control EngPract 7(11):1413–1419
[11] Papoulis A, Pillai SU (2002) "Probability random variables, and stochastic processes". McGraw-Hill, New York
[12] Shieh JI, Wu HH, Huang KK (2010) "A DEMATEL method in identifying key success factors of hospital service quality". Knowledge-Based Syst 23(3):277–282
[13] Tamura H, Wu HH, Huang KK (2010) A DEMATEL method in identifying key success factors of hospital service quality. Knowledge-based syst 23(3): 227-282
[14] Tamura H, Okanishi H, Akazawa K (2006) "Decision support for extracting and dissolving consumers' uneasiness over foods using stochastic DEMATEL". J Telecommun Inform Technol 4:91–95
[15] Tseng ML (2009) A causal and effect decision making model of service quality expectation using grey-fuzzy DEMATEL approach. Expert SystAppl 36(4):7738–7748
[16] Tzeng GH, Chen WH, Yu RC, Shih ML (2010) "Fuzzy decision maps: a generalization of the DEMATEL methods". Soft Comput 14(11):1141–1150
[17] Victor Devadoss A, Felix A (2013), "A Fuzzy DEMATEL approach to study cause and effect relationship of youth violence" International Journal of Computing Algorithm, Vol 02, , pp, 363-372
[18] Wei GW (2009) "Uncertain linguistic hybrid geometric mean operator and its application to group decision making under uncertain linguistic environment". Int J Uncertain Fuzz 17(2):251–267
[19] Wei-LanSuo, Bo Feng, Zhi-Ping Fan (2012) "Extension of the DEMATEL method in an uncertain linguistic Environment" soft computing 16:471–483
[20] Xu ZS (2004) "Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment". Inform Sci 168(1–4):171–184
[21] Xu ZS (2006) "An approach based on the uncertain LOWG and induced uncertain LOWG operators to group decision making with uncertain multiplicative linguistic preference relations". Decis Support Syst 41(2):488–499
[22] Xu YJ, Da QL (2008) "A method for multiple attributes decision making with incomplete weight information under uncertain linguistic environment". Knowledge-Based Syst21(8):837–841
[23] Zadeh LA (1965) "Fuzzy sets". Inform Control 8(3):338–353
[24] Zadeh LA (1975) "The concept of a linguistic variable and its application to approximate reasoning" (Part II). Inform Sci 8:301–357