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New Expression to Mass Transfer Coefficient in a Wetted-Wall Column

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Abstract : *The mass transfer from a falling liquid film to gas steam is the subject of interest to many investigators. Wetted wall columns are generally used for this kind of studies. Basically this kind of transfer process involves simultaneous heat and mass transfer. However, the temperature of the falling liquid is kept constant when operated as an adiabatic humidifier such device is useful in mass transfer studies. In this work, a new analytical expression is derived for the mass transfer coefficient. The equation developed in this study and existing mass transfer equations were evaluated with literature data.*

Keywords: *Adiabatic humidifier, Mass transfer coefficient ,Water-air system, Wetted wall column*

1. Introduction

Wetted wall columns have found applications in the absorption process [1]. In calculating the size of an absorber the important factor is the value of the mass transfer coefficient or the height of the transfer unit. Whilst the total flow rates of the gas and liquid streams will be fixed by the process, it is necessary to fix the most suitable flow rates per unit area through the column [2-4]. The gas flow rate is limited by the fact that the flooding rate must not be exceeded and there will be a serious in the performance if the liquid flow rate is very low [5]. Large surface is available for heat and mass transfer in this type of equipment because of the low mass transfer rate inherent in wetted wall equipment. It is convenient to examine the effects of flow rates of the gas and liquid on transfer coefficients and also to investigate the influence of variables such as temperature, pressure and diffusivity [5-9]. In laboratory wetted wall column have been used by a number of investigators [2-9] and they have proved valuable in determining the importance of the various factors, and have served as a basis from which correlations have been developed for packed towers.

In this work a theoretical analysis of the wetted wall column has been presented. A new analytical expression for mass transfer coefficient has been presented.

2. Theory

Existing model [1]. The mass transfer from a falling film to gas steam is the subject of interest to many investigators. Basically this kind of transfer process involves simultaneous heat and mass. However, the temperature of the liquid may be kept constant in the wetted wall column when operated as an adiabatic humidifier (i.e., there is no fugacity gradient in the liquid phase). Under these conditions the transfer process takes place only through the gas phase, and this kind of devise is well suited to a study of mass transfer in the gas phase. A typical wetted wall column is shown is Fig. 1. The gas phase transfer coefficient k_G as defined in Eq. (1) is calculated from experimental data by integrating Eq. (2) over the measured conditions of the transfer section of the wetted wall column.

For a differential element of interfacial area d*S*, measuring *S* from the bottom of the transfer section

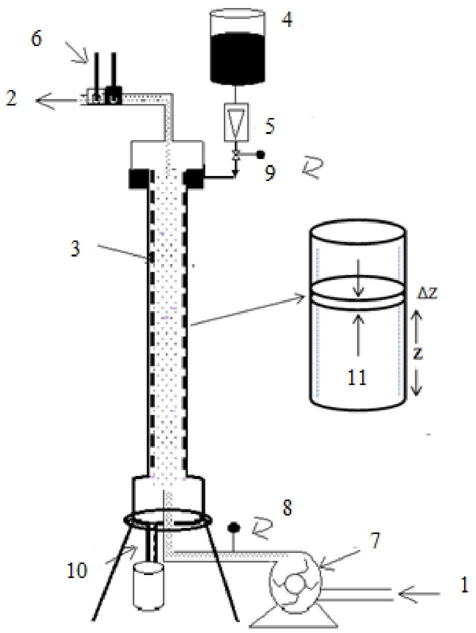

Fig. 1. Typical wetted wall column: *(1)* Inlet air, *(2)* gas with water vapour at outlet, *(3)* liquid film without ripples, *(4)* liquid reservoir tank, *(5)* rotameter, *(6)* dry bulb and wet bulb, *(7)* air blower, *(8)* valve for blower, *(9)* valve for liquid inlet, *(10)* liquid drain, *(11)* fully developed section without ripples

$$\bar{N}_A = k_G\, S\,(-\Delta f) \tag{1}$$

Where

$\bar{N}_A$ = mass (or moles) of component transferred per unit time

k_G = gas phase mass transfer coefficient, mass (or moles) per unit time, unit drop in fugacity, and unit cross section area.

$-(\Delta f)$ = Decrease in fugacity of the transferring constituent

$$V_s\, dY_A = k_G\,(-\Delta f)\, dS \tag{2}$$

Where

k_G = gas phase mass transfer coefficient kgmol /(hr m^2 atm).

V_s = Compound A free gas rate of flow in kgmol /(m^2 hr).

S = the interfacial area , m^2.

Y= gas composition (mole of compound A/mole compound A free gas).

$-(\Delta f) = f_{Ai} - f_{Ag}$ = fugacity of transferring component at interface minus its fugacity in the gas phase.

Since

$$dY_A = \frac{dy_A}{\left(1-y_A\right)^2} \tag{3}$$

and

$$V = \frac{V_s}{1-y_A} \tag{4}$$

Where

y_A = gas composition (i.e., mole fraction of component A)

V = gas rate of flow (moles of total gas per hr)

Substituting Eqs. (3) and (4) in Eq. (2) and rearranging

$$\int_{y_{A1}}^{y_{A2}} \frac{dy_A}{\left(1-y_A\right)\left(f_{Ai}-f_{Ag}\right)} = \int_0^S \frac{k_G}{V} dS \tag{5}$$

The left hand side of the Eq. (5) approximately becomes $k_G S/V$ when gas phase mass transfer coefficient is proportional to the total mass rate of gas flow. This is further justified most practical cases the variation in V is small through the transfer section. In the integration of left-hand side of the Eq. (5), the fugacity of the transferring component at the interface f_i is a constant and may be evaluated at low pressures as the vapour pressure of the liquid at the adiabatic saturation temperature. Thus the integrated form of Eq. (5) is

$$\int_{y_{A1}}^{y_{A2}} \frac{dy_A}{\left(1-y_A\right)\left(p_t y_{Ai}-p_t y_A\right)} = \int_0^S \frac{k_G}{G} dS \tag{6}$$

The Eq. (6) left hand side integrand is partial fractioned as follows

$$\frac{1}{(1-y_A)\,(y_{Ai}-y_A)} = \frac{K_1}{(1-y_A)} + \frac{K_2}{(y_{Ai}-y_A)}$$

Constant comparison

$$1 = K_1\left(y_{Ai}\right) + K_2 \tag{7}$$

y_A Coefficients comparison

$$0 = -K_1 - K_2 \tag{8}$$

Solving Eq. (4) and Eq. (5) we get K_1 and K_2

$$K_1 = \frac{-1}{1-y_{Ai}}$$

$$K_2 = \frac{1}{1-y_{Ai}}$$

Then the Eq. (6) is integrated between y_{A1} and y_{A2} to get gas side film mass transfer coefficient is

$$k_G S = \frac{V}{(y_{Ai}-1)p_t} \ln\left(\frac{(1-y_{A1})}{(1-y_{A2})} \cdot \frac{(y_{Ai}-y_{A2})}{(y_{Ai}-y_{A1})}\right) \tag{9}$$

$$k_G = \frac{V}{S(y_{Ai}-1)p_t} \ln\left(\frac{(1-y_{A1})}{(1-y_{A2})} \cdot \frac{(y_{Ai}-y_{A2})}{(y_{Ai}-y_{A1})}\right) \tag{10}$$

$$k_G = \frac{V}{S(1-y_{Ai})p_t} \ln\left(\frac{(1-y_{A2})}{(1-y_{A1})} \cdot \frac{(y_{Ai}-y_{A1})}{(y_{Ai}-y_{A2})}\right) \tag{11}$$

For many cases the gas mixture contains only a small amount of solute, the value of y become negligible compared to 1.0 and Eq. (9) may be simplified to

$$k_G = \frac{V}{Sp_t} \ln\left(\frac{(y_{Ai}-y_{A1})}{(y_{Ai}-y_{A2})}\right) \tag{12}$$

Eq. (12) is arranged as follows

$$k_G = \frac{V}{Sp_t} \cdot \frac{(y_{A2}-y_{A1})}{\dfrac{(y_{Ai}-y_{A1})-(y_{Ai}-y_{A2})}{\ln\left(\dfrac{y_{Ai}-y_{A1}}{y_{Ai}-y_{A2}}\right)}} \tag{13}$$

$$k_G = \frac{V}{S} \cdot \frac{(y_{A2}-y_{A1})}{(p_{Ai}-\bar{p}_A)_{LM}} \tag{14}$$

2.1 New model. Solute balance over the differential element gives the following equation

$$\frac{\pi}{4} d^2 G y_A \bigg|_{z+\Delta z} - \frac{\pi}{4} d^2 G y_A \bigg|_{z} = N_A a \Delta V = N_A a \frac{\pi}{4} d^2 \Delta z \tag{15}$$

From the first principle of calculus Eq. (15) is written in differential form as

$$\frac{d(Gy_A)}{dZ} = N_A \, a \tag{16}$$

Where *a* is known as specific gas-liquid interfacial area.

$$a = \frac{area}{volume} = \frac{\pi d Z}{\dfrac{\pi d^2}{4} Z} = \frac{4}{d}$$

Eq. (16) is modified using Eq. (4) as

$$\frac{d\left(G_s \left({}^{y_A}\!\big/_{1-y_A} \right) \right)}{dZ} = N_A\, a \qquad (17)$$

Eq. (17) separated in terms of variables as

$$G_s \frac{dy_A}{(1-y_A)^2} = N_A\, a\, dZ \qquad (18)$$

Eq. (18) is reduced as

$$G_s \frac{dy_A}{(1-y_A)^2} = N_A \frac{4}{d} dZ \qquad (19)$$

Where

$N_A = k_G\left(\bar{p}_{Ai} - \bar{p}_A\right)$ is the molar flux of component A

$\bar{p}_A$ is partial pressure of the component A in gas stream (i.e., $y_A p_t$)

$\bar{p}_{Ai}$ is partial pressure of the component A in interface (i.e.,$y_{Ai} p_t$)

NOTE: It's very important to know $k_G \bar{p}_{B,M} = k'_G p_t$ where (k_G mass transfer coefficient for equimolal counter diffusion; k'_G mass transfer coefficient for the case diffusion of "A" through nondiffusing "B"). However, $k_G = k'_G$ when $\bar{p}_{B,M} \approx p_t$ and it is true for very dilute systems.

$$G_s \frac{dy_A}{(1-y_A)^2} = k_G\, p_t \left(y_{Ai} - y_A\right) \frac{4}{d} dZ \qquad (20)$$

If we assume mass transfer coefficient is constant throughout the length then we can integrate Eq. (20) as follows

$$\int_{y_{A1}}^{y_{A2}} \frac{dy_A}{(1-y_A)^2\left(y_{Ai} - y_A\right)} = \int_0^Z \frac{k_G\, p_t\, 4}{G_s d} dZ = \frac{4 k_G\, p_t Z}{G_s d} \qquad (21)$$

The left hand side integrand is partial fractioned as follows

$$\frac{1}{(1-y_A)^2\left(y_{Ai} - y_A\right)} = \frac{K_3}{(1-y_A)} + \frac{K_4}{(1-y_A)^2} + \frac{K_5}{\left(y_{Ai} - y_A\right)}$$

$$1 = K_3(1-y_A)(y_{Ai} - y_A) + K_4(y_{Ai} - y_A) + K_5(1-y_A)^2$$

Constant comparison

$$1 = K_3\left(y_{Ai}\right) + K_4\left(y_{Ai}\right) + K_5 \qquad (22)$$

y_A Coefficients comparison

$$0 = K_3(-1 - y_{Ai}) + K_4(-1) + K_5(-2) \tag{23}$$

y_A^2 Coefficients comparison

$$0 = K_3 + K_4 \tag{24}$$

Therefore

The Eq. (22) is rewritten as

$$1 = K_3(y_{Ai} - 1) + K_4(y_{Ai}) \tag{25}$$

The Eq. (23) is rewritten as

$$0 = K_3(1 - y_{Ai}) - K_4 \tag{26}$$

Solving Eq. (25) and Eq. (26) we get K$_3$, K$_4$ and K$_5$ as follows

$$K_3 = \frac{-1}{(1-y_{Ai})^2}\ ;\ K_4 = \frac{-1}{(1-y_{Ai})}\ ;\ K_5 = \frac{1}{(1-y_{Ai})^2}$$

Then the Eq. (21) is integrated as follows

$$\int_{y_{A1}}^{y_{A2}} \frac{-dy_A}{(1-y_{Ai})^2(1-y_A)} + \int_{y_{A1}}^{y_{A2}} \frac{-dy_A}{(1-y_{Ai})(1-y_A)^2} + \int_{y_{A1}}^{y_{A2}} \frac{dy_A}{(1-y_{Ai})^2(y_{Ai}-y_A)} = \frac{4k_G p_t Z}{G_s d} \tag{27}$$

$$\frac{1}{(1-y_{Ai})^2}\ln\left(\frac{1-y_{A2}}{1-y_{A1}}\right) + \frac{1}{(1-y_{Ai})}\left\{\frac{1}{(1-y_{A2})} - \frac{1}{(1-y_{A1})}\right\} - \frac{1}{(1-y_{Ai})^2}\ln\left(\frac{y_{Ai}-y_{A2}}{y_{Ai}-y_{A1}}\right) = \frac{4k_G p_t Z}{G_s d}$$
(28)

Therefore, gas side film mass transfer coefficient is

$$k_G = \left[\frac{G_s d}{4Zp_t}\right]\left\{\frac{1}{(1-y_{Ai})^2}\ln\left\{\frac{(1-y_{A2})(y_{Ai}-y_{A1})}{(1-y_{A1})(y_{Ai}-y_{A2})}\right\} + \frac{1}{(1-y_{Ai})}\left\{\frac{y_{A2}-y_{A1}}{(1-y_{A2})(1-y_{A1})}\right\}\right\} \tag{29}$$

Eq. (29) further modified as volumetric mass transfer coefficient as

$$k_G a = \left[\frac{G_s}{Zp_t}\right]\left\{\frac{1}{(1-y_{Ai})^2}\ln\left\{\frac{(1-y_{A2})(y_{Ai}-y_{A1})}{(1-y_{A1})(y_{Ai}-y_{A2})}\right\} + \frac{1}{(1-y_{Ai})}\left\{\frac{y_{A2}-y_{A1}}{(1-y_{A2})(1-y_{A1})}\right\}\right\} \tag{30}$$

3. Results and Discussions

To validate the present model, the experimental results of R.C. Cairns and G. H. Roper [8] has been considered. The wetted wall column employed was a glass tube 0.02256 m I.D × 0.94616 m long. The reported dry gas flow rates, inlet humidity, out let humidity and water temperature are considered. The inlet and outlet humidity values were converted to mole fraction units. The relation between mole faction and humidity is

$$y = \frac{H/M_A}{1/M_B + H/M_A} \tag{31}$$

Where H is the humidity in kg water vaour/kg dry air; M_A and M_B are molar mass of water and air.

Table 1 represents the experimental values. Table 2 represents the mass transfer coefficients estimated from the Eqs. (11), (12) and (29). From the results it is clearly evident that the all the models estimate the mass transfer coefficient differently. When the rate of mass transfer is fixed the product of mass transfer coefficient, area of the mass exchanger and the average driving force will be fixed. Hence the area will be less for a given configuration when higher mass transfer coefficient is used. Therefore, the higher the value for the mass transfer coefficient the area required would be lesser and it would be better for the sizing the equipment's. From Table 2, the new model represents the highest mass transfer coefficient hence it may be used to sizing. The mass transfer coefficients estimated follows the order Eq. (12) < Eq. (11) < Eq. (29).

Table 1. Experimental values from Ref. [8]

S.No	V_s	G_s	T_w	y_{A1}	y_{A2}	$^*y_{Ai}$
1	0.0613	149.2	357.52	0.481	0.532	0.557
2	0.1097	267.3	352.19	0.345	0.413	0.450
3	0.2701	658.4	352.80	0.352	0.407	0.461
4	0.1819	443.5	356.46	0.440	0.495	0.534
5	0.1207	294.3	360.24	0.541	0.583	0.619
6	0.1388	338.3	365.46	0.697	0.726	0.755
7	0.1897	462.5	346.52	0.236	0.315	0.355
8	0.2723	663.7	343.63	0.180	0.259	0.314
9	0.3801	926.5	339.96	0.138	0.216	0.268
10	0.0294	71.8	369.19	0.827	0.847	0.866
11	0.0350	85.4	367.85	0.80	0.822	0.825
12	0.0455	110.9	366.91	0.757	0.789	0.796
13	0.0520	126.8	366.46	0.733	0.768	0.783
14	0.0746	181.9	364.07	0.658	0.698	0.716
15	0.0891	217.2	362.74	0.618	0.664	0.681
16	0.0816	198.9	363.80	0.638	0.686	0.709
17	0.0676	164.8	364.85	0.678	0.721	0.738
18	0.1042	254.0	361.57	0.58	0.629	0.651
19	0.0984	239.9	362.30	0.594	0.643	0.670
20	0.1199	292.4	311.96	0.011	0.051	0.068
21	0.1201	292.8	312.02	0.012	0.055	0.069
22	0.1201	292.8	311.80	0.013	0.053	0.068
23	0.1202	293.1	312.02	0.012	0.055	0.069
24	0.1204	293.5	311.74	0.012	0.053	0.068
25	0.1048	255.5	310.07	0.009	0.048	0.062
26	0.1791	436.7	312.30	0.013	0.050	0.070
27	0.3749	913.9	305.30	0.009	0.033	0.047

*Estimated using $y_{Ai} = \bar{p}_A / p_t$

where $\bar{p}_A$ = exp(73.649-7258.2/T-7.3037ln(T)+4.1653×10^{-5}(T)^2) Pa [10]

Table 2. Estimated Mass Transfer coefficient from Eqs. (11), (12) and (29)

S.No	Eq. (11)	Eq. (12)	Eq. (29)
1	3.9098	1.9109	4.9956
2	4.2336	2.6011	5.5606
3	6.9022	4.2567	9.3324
4	7.7596	4.0976	10.3969
5	6.8633	2.9844	9.2798
6	16.8629	4.8298	23.8446
7	5.4249	3.8896	7.0702
8	5.5887	4.3251	7.4367
9	7.3414	5.9933	9.5303
10	10.9666	1.7753	16.6389
11	31.8910	5.8973	38.2825
12	20.2482	4.5103	26.2334
13	13.9095	3.4156	19.1178
14	11.9965	3.8088	15.9929
15	12.6951	4.4922	16.7109
16	11.2575	3.7495	15.7462
17	13.5321	3.9935	18.4043
18	10.8661	4.2389	14.4548
19	9.7564	3.6808	13.4819
20	2.2485	2.1691	2.4692
21	2.6454	2.5433	2.8937
22	2.4489	2.3570	2.6742
23	2.6404	2.5381	2.8864
24	2.4794	2.3872	2.7093
25	2.1108	2.0415	2.2943
26	2.9458	2.8424	3.2362
27	5.3816	5.2566	5.7317

4. Conclusions

The mass transfer coefficient obtained using the new model is more accurate than the existing models. Therefore, this model may be used for sizing of equipment. The order of magnitude for mass transfer coefficient for air and water system based on this model is about 10^{-3} kgmol / (s m^2 atm) whereas the order of magnitude for the volumetric mass transfer coefficient is 0.1 – 1.8 kgmol / (s m^3atm).

NOTATION

a- specific surface of wetted wall column, m^2/m^3
B- non diffusing component, air
d – diameter of the wetted wall column, m
f_{Ai} – fugacity of component A at interface , atm
f_{Ag} – fugacity in the gas phase, atm
G – gas mass velocity in new model, kgmol/(m^2 hr)
G_S- dry gas mass velocity in new model, kgmol/(m^2 hr)
H- humidity, kg water vapour/kg dry air
k_G- gas side mass transfer coefficient kgmol/(m^2hr atm) for equimolar counter diffusion of A and air
k'_G- gas side mass transfer coefficient kgmol/(m^2 hr atm) for the case diffusion of "A" through non diffusing air
K_1-constant in existing model
K_2-constant in existing model
K_3-constant in new model
K_4-constant in new model
K_5-constant in new model
$k_G\,a$- volumetric mass transfer coefficient, kgmol/(hr m^3 atm)
M_A- molar mass of water vapour, 18.02 g/mol
M_B-molar mass of dry air, 28.89 g/mol
N_A- molar flux of the component A in new model, kgmol/(m^2 hr)
$\bar{N}_A$ - rate of mass transfer , mass (or moles) /hr
$\bar{p}_A$ - partial pressure of the component A in gas phase, atm
$\bar{p}_{Ai}$ - partial pressure of component A at interface, atm
$\bar{p}_{BM}$ - log mean pressure difference of component B in new model
p_t - total pressure of the column, atm
S – interfacial area of the wetted wall column in exiting model, m^2
T- temperature of the water
V- gas rate of flow in exiting model , kgmol/hr
V'- dry gas rate of flow in existing model, kgmol/hr
y_A- mole fraction of component A in gas phase
y_{Ai}- mole fraction of component A at interface
y_{A1} - mole fraction of A in inlet air
y_{A2} - mole fraction of A in outlet air
Δ - difference
SUBSCRIPTS AND SUPERSCRIPTS
A – component A which is water
Ai- component A at interface
Ag- component at gas phase
s- non-transfer component which is air
w- water temperature
1- wetted wall column inlet
2- wetted wall column outlet
$'$ - diffusion of A through non diffusing air

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