Integrated Intelligent Research (IIR)
International Journal of Computing Algorithm
Volume: 06 Issue: 01 June 2017 Page No.27-30
ISSN: 2278-2397

A Study on Graceful and ∅-Graceful Labeling of Some Graphs

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Abstract – In this paper, we prove that result on graceful labeling of path and symmetrical trees are graceful graphs. We also prove that result on ϕ-graceful labeling of paths, flower graph, friendship graph.

Keywords: Path, symmetrical trees, flower graph, friendship graph

I. INTRODUCTION

Graph labeling is an active area of research in graph theory. Graph labeling where the vertices are assigned some value subject to certain condition. Labeling of vertices and edges play a vital role in graph theory. To begin with simple, finite, connected and undirected graph $G = (V(G), E(G))$ with $|V(G)| = p$ and $|E(G)| = q$. For standard terminology and notation we follow Gallian [1], Gross and Yellen [2]. The definitions and other information which serve as prerequisites for the present investigation.

II. DEFINITIONS

Definition 2.1: A function f is called *graceful labeling* of a graph G if $f: V(G) \to \{0, 1, \dots, q\}$ is injective and the induced function

$f^*: E(G) \to \{0, 1, \dots, q\}$ defined as

$$f^*(e = uv) = |f(u) - f(v)|$$

is bijective. The graph which admits graceful labeling is called a graceful graph.

Definition 2.2: A rooted tree in which every level contains vertices of the same degree is called *symmetrical trees*. The following result is a gracefully labeled symmetrical tree on 15 vertices.

Definition 2.3: A function f is called a *∅-graceful labeling* of graph G, if $f: V(G) \to \{0, 1, \dots, n-1\}$ is injective and the induced function $f^*: E(G) \to N$ is defined as $f^*(e = uv) = 2\{f(u) + f(v)\}$, then edge labels are distinct. The concept of graceful labeling is an active area of research in graph theory which has rigorous applications in coding theory, communication networks, optimal circuits layouts and graph decomposition problems. For a dynamic survey of various graph labeling problems along with an extensive bibliography we refer to Gallian [1].

The following three types of problems are considered generally in the area of graph labeling.

1. How particular labeling is affected under various graph operations;
2. Investigation of new families of graphs which admit particular graph labeling;
3. Given a graph theoretic property and characterizing the class/classes of graphs with property that admits particular graph labeling.

It is clear that the problems of second type are largely studied than the problems of first and third types. The present work is aimed to discuss some problems of the first kind in the context of graceful and ∅-graceful labeling.

Definition 2.4: Let G be a graph with order 'n' and size $n-1$, such that exactly one node is adjacent to every other $n-1$ nodes, this is also known as *flower graph*. The resulting graph is flower graph with $n-1$ petal.

Definition 2.5: A *friendship graph* F_n is a graph which consists of n triangles with common vertices.

III. RESULTS ON GRACEFUL LABELING

Theorem 3.1: Every path (P_n) graph is graceful graphs.

Proof: Let G be a path graph.
Label the first vertex 0, and label every other vertex increasing by 1 each time.

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Label the second vertex q and label every other vertex decreasing by 1 each time.
There are $q+1$ vertices,
so the first set will label it's vertices with numbers from the set $\{0,1,\dots,q/2\}$ if q is even and from the set $\{0,1,\dots,(q+1)/2\}$ if q is odd.
The second set will label it's vertices with numbers from the set
$\{(q+2)/2,\dots,q\}$ if q is even and $\{(q+3)/2,\dots,q\}$ if q is odd.
Thus, the vertices are labeled legally.
With the vertices labeled in this manner, the edges attain the values
$q, q-1, q-2, \dots, 1$ in that order.
Thus, this is a graceful labeling, so G is graceful.
∴, All path P_n graphs are graceful.

Illustration:
Graceful labeling of the graph P_4 is shown in fig.1.

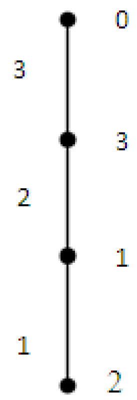

Fig1: Graceful graph of P_4

Theorem 3.2: All symmetrical trees are graceful.

Proof: We prove the theorem by induction method.
The number of layers that all symmetrical trees are graceful, and there exists a graceful labeling which assigns the number 1 to the root.

If 'T' is a symmetrical tree with 0 layers,
Then, it consists of 0 edges and just one vertex.
And clearly, there is a graceful which assigns 1 to the vertex.
Suppose, we have proved that for some
$l > 0$.
All symmetrical trees with $\leq l-1$ layers are graceful and each of them has a graceful which assigns the number 1 to the root.

$$\Rightarrow f(v) = 1$$
This completes the induction.
∴, All symmetrical trees are graceful.

Illustration:
Gracefully labeled symmetrical tree on 15 vertices is shown in fig.2.

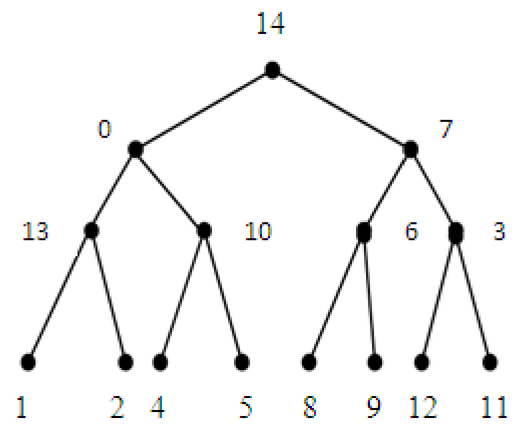

Fig2: Graceful labeling of symmetrical tree on 15-vertex.

IV. RESULTS ON ∅-GRACEFUL LABELING

Theorem 4.1: Path (P_n) graphs are ∅-graceful graphs.

Proof: Let P_n be a path with vertex set $v_1, v_2, v_3, \dots, v_n$ and $e_1, e_2, e_3, \dots, e_{n-1}$ be the edges of the path.
We define the vertex labeling
$f : V(G) \rightarrow \{0,1,2,\dots,n-1\}$
$f(v_1) = 0$
$f(v_2) = 1$
:
:
$f(v_n) = n-1$, vertex labeling can be done in both directions.
The edge labeling function f^* is defined as follows
$f^* : E(G) \rightarrow N$ is defined by
$f^*(uv) = 2\{f(u) + f(v)\}$
$f^*(vv_n) = 2\{f(v) + f(v_n)\}$
:
:
$= \{2, 6, 10, \dots, (4n-6)\}$ such that the edge labels are distinct.
In view of above labeling pattern the paths are ∅-graceful labeling.
Hence P_n is a ∅-greceful graph.

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Illustration:

∅-Graceful labeling of the graph P_4 is shown in fig.3.

Theorem 4.2: Flower graphs are ∅-graceful graphs.

Proof: Let G be a flower graph. Then G has n vertices and (n-1) edges.

Therefore $V = \{v_1, v_2, \dots, v_n\}$ and

$E = \{e_1, e_2, \dots, e_n\}$

We define the vertex labeling

$f : V(G) \rightarrow \{0, 1, 2, \dots, n\text{-}1\}$

$f(v_1) = 0$

$f(v_2) = 1$

:

:

$f(v_n) = n - 1$ such that labeling of the vertices may be clockwise or anticlockwise.

The edge labeling function f^* is defined as follows

$f^* : E(G) \rightarrow N$ is defined by

$f^*(u\,v) = 2\{f(u) + f(v)\}$

$f^*(v_1 v_n) = 2\{f(v_1) + f(v_n)\}$

:

:

$= \{2, 4, 6, 8, \dots, 2n\}$ then the edge labels are distinct and it's in increasing order.

Hence all flower graphs are ∅-graceful graph.

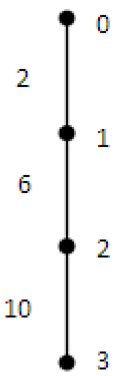

Fig3: ∅ - Graceful graph of P_4.

Illustration:

Flower graph with 8 petals in fig.4.

Theorem 4.3: A Friendship graph F_2 is a ∅-graceful graph.

Proof: Let $G = F_n$ be the Friendship Graph.

Let $v_1, v_2, v_3, \dots, v_n$ be the vertices of F_n and edges are $e_1, e_2, e_3, \dots, e_m$.

We define the vertex labeling

$f : V(G) \rightarrow \{0, 1, 2, \dots, n - 1\}$

$f(v_1) = 0$

$f(v_2) = 1$

:

:

$f(v_n) = n - 1$

The edge labeling function f^* is defined as follows

$f^* : E(G) \rightarrow N$ is defined by

$f^*(uv) = 2\{f(u) + f(v)\}$

$f^*(v_1 v_2) = 2\{f(v_1) + f(v_2)\} = 2$;

:

:

$= \{2, 4, 6, 10, 14, 12\}$ then the edge labels are distinct.

In view of the above labeling pattern the F_2 is a ∅-graceful labeling. Hence F_2 is a ∅-graceful graph.

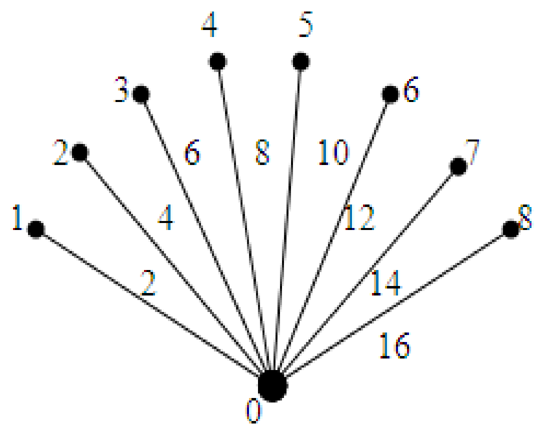

Fig4: ∅ - Graceful graph of flower graph.

Illustration:

∅ - Graceful labeling of the graph F_2 is shown in fig.5.

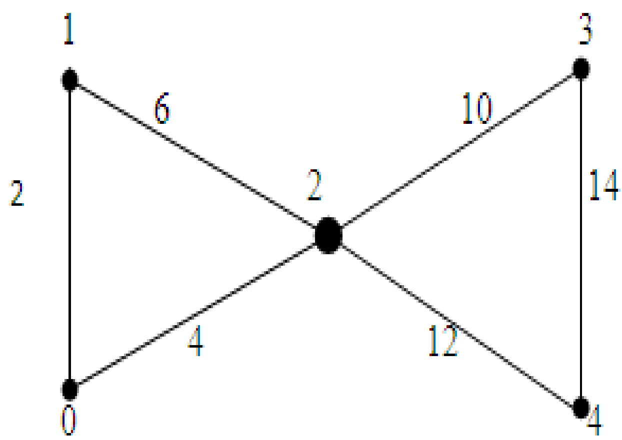

Fig5: ∅ - Graceful graph of F_2 graph.

International Journal of Computing Algorithm
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V. CONCLUSION

In this paper we have shown that every path graphs are Graceful and ∅ - Graceful graph, Friendship graph is a ∅ - Graceful graph, every flower graph is a ∅ - Graceful graph. Are investigated it can also verified for some graphs.

Acknowledgment
We would like to give the grateful thanks to Dr. Seenuvasan and Dr. Rishivarman who gives for their valuable suggestions and improvements to the paper of research project and work. Moreover, we want to thank Parent's, Friends, ICMA17 – GT165 and IJCTA for their encouragement and support.

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