Integrated Intelligent Research (IIR)

International Journal of Computing Algorithm
Volume: 06 Issue: 01 June 2017 Page No.23-26
ISSN: 2278-2397

The Minimum Mean Dominating Energy of Graphs

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Abstract: The energy of a graph was introduced by I.Gutman [4] and its bound were obtained by .Balakrishnan [9].Subsequently the Minimum Dominating Energy was introduced by [8] R.Rajeshkanna.Then Mean Energy (M - energy) was introduced by LauraBuggy, Amaliailiuc, KatelynMccall, and Duyguyen [7]. In this paper, we studied the Mean Domination Energy for some standard graphs and obtained its bounds.

Keywords—Energy of a graph, dominating energy, mean dominating energy

I. INTRODUCTION

The concept of energy of a graph was introduced by I.Gutman [4] in the year 1978,throughout this paper G will denote a simple undirected graph with n vertices. For details, refer the book [1] A subset S of the vertex set $V(G)$ is called Dominating set, if every vertex not in S (every vertex in (V-S)) is adjacent to at least one member of S. The minimal cardinality of such set S is called as Minimum dominating set of G. Let D be the minimum dominating set for a graph $G = (V, E)$, then the minimum dominating matrix of G is a $n \times n$ matrix

$$A_D(G) = \begin{cases} 1 & if \ <v_i, v_j> \in E \\ 1 & \text{if } i=j, v_j \in D \\ 0 & \text{otherwise} \end{cases}$$

Then Eigen values of **A_D(G)** are positive real and symmetric.

Minimum Dominating Energy (MDE):
The minimum dominating energy [8] of graph is given by the sum of absolute values of Eigen values of the minimum dominating matrix.

Minimum Mean Dominating Energy (MMDE): The mean dominating energy of G is defined as $E_D^M(G) = \sum_{i=1}^{n} |\mu_i - \bar{\mu}|$, where $\bar{\mu}$ is the average of the Eigen values $\mu_1, \mu_2, ..., \mu_n$ of the minimum dominating matrix.

Complete Graph: A graph K_n is complete if every pair of vertices are connected by an edge.

Star Graph: A star graph $K_{1,n-1}$ of order n is a tree on n nodes with one node is having vertex degree *(n-1)* and all the other *(n-1)* nodes having vertex degree 1.

Cocktail graph: A cocktail graph [1] S_k is the graph on k = 2p vertices u_i , i = 1,2,3,...,k with u_i is non-adjacent to u_{i+p}, for each i= 1,2,...,p. All of the other pairs of vertices are adjacent. Note that this is a (k –2) regular graph.

Crown graph: The crown graph $S_n^0 = (V, E)$,for integer $n \geq 2$,is the graph generated from complete bipartite graph by removing perfect matching.
For the fundamental concepts, definitions and earlier results, refer [1, 2, 3, 5, 6].

II. THE MINIMUM MEAN DOMINATING ENERGY OF SOME STANDARD GRAPHS

Theorem1: For Complete Graph k_n the Minimum Mean Dominating Energy is

$$\frac{(n-2)(n+1)}{n} + \sqrt{n^2 - 2n + 5}. \ (n \geq 2)$$

Proof: K_n is Complete graph on n-vertices, the vertex set is given by $V = \{v_1, v_2, ..., v_n\}$.
The minimum dominating set is a singleton set, let us assume that $D = \{v_1\}$.Then the minimum dominating matrix

$$A_D(k_n) = \begin{pmatrix} 1 & 1 & 1 & & 1 & 1 \\ 1 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 0 & & 1 & 1 \\ & \vdots & & \ddots & & \vdots \\ 1 & 1 & 1 & \cdots & 1 & 0 \end{pmatrix}_{n\times n}$$

The characteristic polynomial is
$| A_D(K_n) - \mu I | = 0$ and computed as:

$$(\mu + 1)^{n-2}(\mu^2 - (n-1)\mu - 1) = 0.$$

The Eigen values are: one is of multiplicity (n-2) and other two are simple roots; they are

$$\mu = -1 \ \ \& \ \ \mu = \frac{(n-1) \pm \sqrt{n^2 - 2n + 5}}{2}.$$

And hence the Mean $\bar{\mu} = \frac{1}{n}$.

Minimum Dominating Energy is

$$E_D(k_n) = |-1|(n-2) + \left|\frac{(n-1) \pm \sqrt{n^2-2n+5}}{2}\right|$$
$$= (n-2) + \sqrt{n^2 - 2n + 5}.$$

International Journal of Computing Algorithm
Volume: 06 Issue: 01 June 2017 Page No.23-26
ISSN: 2278-2397

Minimum Mean Dominating Energy is computed as

$$E_D^M(k_n) = \sum_{i=1}^{n-2} \left| -1-\frac{1}{n}\right| + \left|\frac{(n-1)\pm\sqrt{n^2-2n+5}}{2} - \frac{1}{n}\right|$$

$$= \frac{(n-2)(n+1)}{n} + \left|\frac{n(n-1)\pm n\sqrt{n^2-2n+5}-2}{2n}\right|$$

$$= \frac{(n-2)(n+1)}{n} + \left|\frac{(n^2-n-2)\pm n\sqrt{n^2-2n+5}}{2n}\right|$$

$$= \frac{(n-2)(n+1)}{n} + \sqrt{n^2-2n+5} \blacksquare$$

Theorem 2: For Star Graph $k_{1,n-1}$ the Minimum Mean Dominating Energy is

$$\frac{(n-2)}{n} + \sqrt{4n-3}\,.\ (n \geq 2)$$

Proof: $k_{1,n-1}$ is Star Graph which is a bipartite graph with vertex set $V = \{v_1, v_2, .., v_n\}$.Let us take v_1 be the vertex in the one set and remaining $(n-1)$ vertices are in the other set. Hence, the minimum dominating set is $D = \{v_1\}$.

Then the minimum dominating matrix

$$A_D(k_{1,n-1}) = \begin{pmatrix} 1 & 1 & 1 & & 1 & 1 \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & & 0 & 0 \\ & \vdots & & \ddots & & \vdots \\ 1 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}_{n\times n}$$

Then the characteristic polynomial is given by

$|A_D(k_{1,n-1}) - \mu I| = 0$ is $\mu^{n-2}(\mu^2 - \mu - (n-1)) = 0$.

Then Eigen values are $\mu = 0$ is of multiplicity (n-2) and the other two are simple roots $\mu = \frac{1\pm\sqrt{4n-3}}{2}$. and hence Mean $\bar{\mu} = \frac{1}{n}$.

Minimum Dominating Energy is computed as

$$E_D(k_{1,n-1}) = \sum_{i=1}^{n-2} |0|\,(n-2) + \left|\frac{1\pm\sqrt{4n-3}}{2}\right| = \sqrt{4n-3}.$$

Minimum Mean Dominating Energy is computed as:

$$E_D^M(k_{1,n-1}) = \sum_{i=1}^{n-2} \left|0-\frac{1}{n}\right| + \left|\frac{1\pm\sqrt{4n-3}}{2} - \frac{1}{n}\right|$$

$$= \frac{(n-2)}{n} + \left|\frac{(n-2)\pm n\sqrt{4n-3}}{2n}\right|$$

$$= \frac{(n-2)}{n} + \sqrt{4n-3} \blacksquare$$

Theorem 3: For Cocktail Party Graph S_k the Minimum Mean Dominating Energy is

$$\frac{4n^2-4n-7}{2n} + \sqrt{4n^2-4n+9}.$$

Proof: Let S_k be the Cocktail party graph with vertex set $V = \bigcup_{i=1}^{n}\{u_i, v_i\}$

The minimum dominating set is $D = \{u_1, v_1\}$

Then the minimum dominating matrix $A_D(S_k)$

$$= \begin{pmatrix} 1 & 0 & 1 & 1 & \cdots & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & \cdots & 1 & 1 & 1 & 1 \\ & & & & \vdots & & & & \\ 1 & 1 & 1 & 1 & & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & \cdots & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & \cdots & 1 & 1 & 0 & 0 \end{pmatrix}$$

Then the characteristic polynomial is given by

$|A_D(S_k) - \mu I| = 0$

$$\mu^{n-1}(\mu-1)(\mu+2)^{n-2}(\mu^2 - (2n-3)\mu - 2n) = 0$$

The Eigen values are $\mu = 0$ is a root of multiplicity $(n-1)$, $\mu = 1$ (is a simple root), $\mu = -2$, is a root of multiplicity $(n-2)$ and two simple roots $\mu = \frac{(2n-3)\pm\sqrt{4n^2-4n+9}}{2}$

and hence Mean $\bar{\mu} = \frac{1}{n}$

Minimum Dominating Energy is computed as:

$$E_D(k_{n\times 2}) = |0|\,(n-1) + 1 + |-2|\,(n-2) + \left|\frac{2n-3\pm\sqrt{4n^2-4n+9}}{2}\right|$$

$$= (2n-3) + \sqrt{4n^2-4n+9}$$

Minimum Mean Dominating Energy is computed as:

$$E_D^M(k_{n\times 2}) = \sum_{i=1}^{n-1}\left|0-\frac{1}{n}\right| + \left|1-\frac{1}{n}\right| + \sum_{i=1}^{n-2}\left|-2-\frac{1}{n}\right| + \left|\frac{2n-3\pm\sqrt{4n^2-4n+9}}{2} - \frac{1}{n}\right|$$

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$$= \frac{(n-1)}{n} + \frac{(n-1)}{n} + \frac{(n-2)(2n+1)}{n} + \left| \frac{(2n^2-3n-2) \pm n\sqrt{4n^2-4n+9}}{2n} \right|$$

$$= \frac{2n^2-n-4}{n} + \sqrt{4n^2-4n+9} \;\blacksquare$$

Theorem 4: For Crown Graph S_n^0 ($n \geq 2$) the Minimum Mean Dominating Energy is

$$2(n-2) + \sqrt{n^2-2n+5} + \sqrt{n^2+2n-3}$$

.

Proof : Let S_n^0 be the Crown Graph with vertex set $V = \{ u_1, u_2, \ldots, u_n, \; v_1, v_2, \ldots, v_n \}$.

Then minimum dominating set is $D = \{ u_1, v_1 \}$

Then the minimum dominating matrix $A_D(s_n^0)$

$$= \begin{bmatrix} 1 & 0 & 0 & . & . & 0 & 0 & 1 & 1 & . & . & 1 \\ 0 & 0 & 0 & . & . & 0 & 1 & 0 & 1 & . & . & 1 \\ 0 & 0 & 0 & . & . & 0 & 1 & 1 & 0 & . & . & 1 \\ . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & 0 & 1 & 1 & 1 & . & . & 0 \\ 0 & 1 & 1 & . & . & 1 & 1 & 0 & 0 & . & . & 0 \\ 1 & 0 & 1 & . & . & 1 & 0 & 0 & 0 & . & . & 0 \\ 1 & 1 & 0 & . & . & 1 & 0 & 0 & 0 & . & . & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . \\ 1 & 1 & 1 & . & . & 0 & 0 & 0 & 0 & . & . & 0 \end{bmatrix}$$

The characteristic polynomial $| A_D(s_n^0) - \mu I | = 0$ is

$$(\mu-1)^{n-2} (\mu+1)^{n-2} (\mu^2 - (n-1)\mu - 1)(\mu^2 + (n-3)\mu - (2n-3)) = 0$$

Then Eigen values are $\mu = 1$ of multiplicity(n–2), $\mu = -1$ of multiplicity(n–2), and simple roots

$$\mu = \frac{(n-1) \pm \sqrt{n^2-2n+5}}{2},$$

$$\mu = \frac{(3-n) \pm \sqrt{n^2+2n-3}}{2}$$

Mean $\bar{\mu} = \frac{1}{n}$.

Minimum Dominating Energy is computed as

$$E_D(S_n^0) = |1|(n-2) + |-1|(n-2) + \left| \frac{(n-1) \pm \sqrt{n^2-2n+5}}{2} \right| + \left| \frac{(3-n) \pm \sqrt{n^2+2n-3}}{2} \right|$$

$$= 2(n-2) + \sqrt{n^2-2n+5} + \sqrt{n^2+2n-3}.$$

Minimum Mean Dominating Energy is computed as

$$E_D^M(S_n^0) = \sum_{i=1}^{n-2} \left|\left(1 - \frac{1}{n}\right)\right| + \sum_{i=1}^{n-2} \left| -1 - \frac{1}{n} \right| + \left| \frac{(n-1) \pm \sqrt{n^2-2n+5}}{2} - \frac{1}{n} \right| + \left| \frac{(3-n) \pm \sqrt{n^2+2n-3}}{2} - \frac{1}{n} \right|$$

$$= \frac{(n-1)(n-2)}{n} + \frac{(n+1)(n-2)}{n} + \left| \frac{(n^2-n-2) \pm n\sqrt{n^2-2n+5}}{2n} \right| + \left| \frac{(3n-n^2-2) \pm n\sqrt{n^2+2n-3}}{2n} \right|$$

$$= \frac{(n-1)(n-2)}{n} + \frac{(n+1)(n-2)}{n} + \sqrt{n^2-2n+5} + \sqrt{n^2+2n-3}$$

$$= 2(n-2) + \sqrt{n^2-2n+5} + \sqrt{n^2+2n-3}$$

$\blacksquare$

III. BOUNDS OF MINIMUM MEAN DOMINATING ENERGY

Result 1 [8]**:** Let G be a simple graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$, edge set E and $D = \{v_1, v_2, \ldots, v_k\}$ be a minimum dominating set. If $\mu_1, \mu_2, \ldots, \mu_n$ are the Eigen values of minimum dominating matrix $A_D(G)$ then

(i) $\sum_{i=1}^{n} \mu_i = |D|$

(ii) $\sum_{i=1}^{n} \mu_i^2 = 2|E| + |D|$.

Result 2[8]**:** *Let* G be a simple graph with n vertices and m edges. If D is the minimum dominating set and $P = |\det \mathbf{A_D(G)}|$ then

$$\sqrt{(2m+k) + n(n-1)P^{2/n}} \leq E_D(G) \leq \sqrt{n(2m+k)}$$

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Theorem 5: *Let* G be a simple graph with n vertices and m edges. If D is the minimum dominating set then

$$\sqrt{(k+2m)-2|\bar{\mu}|\sqrt{n(2m+k)}} \leq E_D^M(G) \leq \sqrt{n\,((k+2m)+\,n\,\bar{\mu}^2\,)}.$$

Proof:

By Cauchy Schwarz inequality

$$(\sum_{i=1}^{n}|a_i\,b_i|)^2 = (\sum_{i=1}^{n}a_i^{\,2})\,(\sum_{i=1}^{n}b_i^{\,2})$$

Take $a_i = 1$, $b_i = |\mu_i - \bar{\mu}|$ then,

$$[E_D^M(G)]^2 = (\sum_{i=1}^{n}1)\,(\sum_{i=1}^{n}|\,\mu_i - \bar{\mu}|^2\,)$$

$$= (n\ )\,(\sum_{i=1}^{n}|\,\mu_i - \bar{\mu}|^2\,)$$

$$\leq n\ \ (\sum_{i=1}^{n}\mu_i^{\,2} + \sum_{i=1}^{n}\bar{\mu}^2\,)$$

$$= \ n\,(\sum_{i=1}^{n}\mu_i^{\,2} + n\ \ \bar{\mu}^2\ )$$

$$\leq\ n\,((2m+k)+\ n\,\bar{\mu}^2\,)$$

Then $E_D^M(G) \leq \sqrt{n\,((k+2m)+\ n\,\bar{\mu}^2\,)}$

Also $[E_D^M(G)]^2 = (\sum_{i=1}^{n}|\,\mu_i - \bar{\mu}|)^2$

$$\geq \sum_{i=1}^{n}|\,\mu_i - \bar{\mu}|^2$$

$$\geq \sum_{i=1}^{n}|\mu_i|^2\ - 2\,|\bar{\mu}\,|\sum_{i=1}^{n}|\,\mu_i\,|$$

$$\geq (k+2m) - 2\,|\bar{\mu}\,|\sqrt{n\,(k+2m)}$$

Then

$$E_D^M(G) \geq \sqrt{(k+2m)\ -\ 2\,|\bar{\mu}\,|\sqrt{n\,(k+2m)}}$$

Hence the theorem ■

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