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Study of Second Order (Rivlin-Ericksen) MHD Fluid Flow Due to an Impulsively Started Flat Plate Through Porous Medium

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Abstract - present note is the study of second order Rivlin-Ericksen MHD fluid flow due to impulsively started flat plate through porous medium. The unsteady laminar flow of an elastico-viscous fluid suspended with a uniform distribution of non-conducting dust particles in presence of uniform transverse magnetic field fixed relative to the plate which is applied normal to the flow. Laplace transform is used and analytical results for the velocity of fluid, mixed particles and the skin friction at the plate are obtained. The effect of magnetic field parameter (*M*), porosity parameter (*K*) and the mass concentration (*l*) of the mixed particle on the velocity of both the fluid, the particle and on the skin friction are discussed numerically and displayed graphically.

Keywords - MHD fluid flow, porous medium, impulsively started flat plate, second order fluid

AMS No: 76T

I. INTRODUCTION

In past decades study in area of visco-elastic fluid has been of great interest. Thirst areas like petroleum industry, paint industry, sedimentation and crude oil industry are of interest for researchers. We consider unsteady two-phase flow of a laminar, incompressible, viscoelastic, electrically conducting Rivlin–Ericksen fluid past a semi-infinite vertical plate embedded in a uniform porous medium subjected to a uniform transverse magnetic field.

Second order fluids are mixed by some dust particles, ash, air bubbles etc and these are treated as non-Newtonian fluids. Initially many researchers have taken this study in their interest.Stokes [1].Rivlin and Erickesn[2], Happel [3] Gupta[4].Saffman[5] detailed regarding mixing of fluids, Pande[6], Prakash[7], Ahmadi and Manvi [8], Ram and Mishra[9], Soundalgekar[10] ,[11] discuss the flow of a viscous fluid and mass transfer effects on them past an impulsively started infinite isothermal vertical plate.Choubey[12],Sharma, Sunil and Suresh [13] discussed Hall effects on thermal instability of Rivlin–Ericksen fluid. Varshney and Rashmi Kumari [14] have solved the hydro magnetic flow due to accelerated motion of an infinite flat plate in the presence of magnetic field fixed relative to the plate. Rana and Kumar [15] studied Thermal instability of a Rivlin-Ericksen elastic-viscous rotating fluid permeated with suspended particles and variable gravity field in porous medium. Srivastava[16] has discussed on MHD flow of dusty Rivilin–Ericksen fluid due to an impulsively started flat plate. Prakash, Kumar and Dwivedi [17] studied Heat transfer in MHD viscoelastic (Walters' liquid model – B) stratified fluid in porous medium under variable viscosity**.** Ravi Kumar and Raju[18] discussed Combined effects of heat absorption and MHD on convective Rivlin-Ericksen flow past a semi-infinite vertical porous plate with variable temperature and suction.Srivastava and Khare[19] have analysed the flow of dusty Rivlin-Ericksen fluid through porous medium.Singh[20] discussed Stability Of superposed fluids through magnetic field with suspended particles Of different permeability saturated through porous medium. Chakraborty [21] study MHD flow and heat transfer of a dusty viscoelastic liquid down an inclined channel in porous medium under variable viscosity and pressure.

In our consideration mix fluids of multiphase systems and having the nature of plastic are useful for various industry purpose. In view of this the present note is the review of Srivastava and Khare [19] under Boussinesq condition [22] as study of second order Rivlin- Ericksen MHD fluid flow due to an impulsively started flat plate through porous medium. Here we will find analytical solutions for velocity of fluid and mixed dust particles along with skin friction. The results are validate with numerical method.

II. MATHEMATICAL ANALYSIS

Following Boussiesq approximatios [22] for particulate system and basic conditions of mix fluids let an electrically conducting second order Rivlin-Ericksen fluid of viscosity μ and electrical conductivity σ contains a flat plate. Consider the *x*-axis along the plate and *y*-axis normal to it. The equations of motion are

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t}=\vartheta\frac{\partial u}{\partial y}+\left(1+\lambda\frac{\partial}{\partial x}\right)\frac{kN}{\rho}(v-u)-\left(1+\lambda\frac{\partial}{\partial x}\right)\left(\frac{\sigma B}{\rho}+\frac{\vartheta}{K}\right)u \quad (1)$$

And

$$\tau\frac{\partial v}{\partial t}=(u-v) \quad (2)$$

The non-dimensional quantities are

$$t^*=\frac{t}{\tau},\ y^*=\frac{y}{\sqrt{\vartheta\tau}},\ u^*=\frac{u}{U},\ v^*=\frac{v}{U},\ \tau=\frac{m}{k}$$

The dimensionless equation "(1)" and "(2)" are

$$\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t}=\vartheta\frac{\partial u}{\partial y}+l\left(1+\alpha\frac{\partial}{\partial t}\right)(v-u)-Q\left(1+\alpha\frac{\partial}{\partial t}\right)u \quad (3)$$

And

$$\frac{\partial v}{\partial t}=(u-v) \quad (4)$$

Where

$$Q = (M^2 + \frac{1}{K})$$

Magnetic field is fixed relative to the plate the eqn. "(3)"will be modified at time $t = 0^*$, the conducting dusty fluid, the plate and the magnetic field are assumed to be stationary everywhere for all later time the plate and the magnetic field are moving with velocity $u = 1^*$. Because the magnetic field is moving while the conducting dusty fluid is initially at rest. The relative motion must be accounted for the origin of coordinate system is fixed in space. Hence by transformation of coordinates of the eqn. "(3)" becomes

$$\left(1 + \alpha\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = \vartheta\frac{\partial u}{\partial y} + l\left(1 + \alpha\frac{\partial}{\partial t}\right)(v - u) - Q\left(1 + \alpha\frac{\partial}{\partial t}\right)(u - 1) \quad (5)$$

The dimensionless boundary conditions are

$$t = 0, u = 0, v = 0, for\ y > 0 \quad (6)$$

$$t > 0,\ u = 1\ \ at\ \ y = 0,\ u \to finite\ as\ y \to \infty \quad (7)$$

Laplace transforms of u & v are taken and equation"(4)"& "(5)" with "(7)", reduce to

$$\frac{\partial^2\bar{u}}{\partial y^2} - p^2\bar{u} = -\frac{1}{S}Q \quad (8)$$

And

$$\bar{v} = \frac{\bar{u}}{1+S} \quad (S<< 1) \quad (9)$$

Where

$$p^2 = (1 + \alpha S)\frac{lS}{1+S} + Q + S \quad (10)$$

The boundary condition "(7)" reduced to

$$\bar{u} = \frac{1}{S}\ at\ y = 0, \bar{u} \to finite\ \ as\ \ y \to \infty \quad (11)$$

The solution of "(8)"subject to the condition "(11)" is

$$\bar{u} = \left[\frac{1}{S} - \frac{1}{Sp^2}Q\right].e^{-py} - \frac{1}{Sp^2}Q \quad (12)$$

The inversion of "(10)" is tedious and to avoid this, let the t is very large so s is very small then

$$p = [\{1 + l + \alpha Q\}S + Q]^{1/2} \quad (13)$$

Then from "(12)" we get (for large t)

$$\bar{u} = \left[\frac{1}{S} - \frac{1}{S+q}\right] + \frac{1}{S+q}e^{-[\{1+l+\alpha Q\}S+Q]^{1/2}y} \quad (14)$$

By inversion theorem equation "(14)" given

$$u = [1 - e^{-qt}] + e^{-qt}.erf_c\left[\frac{\sqrt{1+l+\alpha Q}}{2\sqrt{t}}\right] \quad (15)$$

Where

$$q = \frac{Q}{1 + l + \alpha Q}$$

Similarly taking inverse transform for large value of v we get from "(9)"

$$v = 1 - \left[\frac{q}{q-1}\right]e^{-t} + \frac{e^{-t}}{q-1}.erf_c\left\{\frac{\sqrt{1+l+\alpha Q}}{2\sqrt{t}}\right\} + \frac{e^{-qt}}{q-1}\left[1 - erf_c\left\{\frac{\sqrt{1+l+\alpha Q}}{2\sqrt{t}}\right\}\right] \quad (16)$$

Equations "(15)" and "(16)" represent the velocities of the dusty conducting elastico-viscous fluid and dust particles for large t when the magnetic field is fixed relative to the plate.

Skin Friction

Shearing stresses at the plate for t>>

$$\left(-\frac{\partial u}{\partial y}\right)_{y=0} = \sqrt{\frac{1+l+\alpha Q}{\pi t}}.e^{-qt} \quad (17)$$

III. RESULTS AND DISCUSSION

The expressions obtained for fluid velocity, mixed particles and skin friction are of class A or exponential order so Laplace Transform is a an appropriate tool for modeling. Numerical calculations are carried out for the velocities of the mixed fluid, dust particles and the drag force at the plate from eqns. "(15)", "(16)" and "(17)" respectively for (large t). Here finite difference "Crank-Nicolson Technique"* is used to validate the results for various values of the parameters (M), (l) and (K) while velocity considered at both ends of the plate .

Figures-(1) to (3) represent the velocity profiles of dusty fluid for different parameters. Figure-(1) shows the variation of velocity u with magnetic parameter M. It is observed that the velocity decreases as M increases. Figure-(2) shows that an increase in porosity parameter K causes an increase in velocity profile of dusty fluid. From Figure-(3), that the velocity of dusty fluid decreases as the mass concentration (l) of the dust particle increases. Figures-(4) to (6) represent the velocity profiles of dust particles for various parameters. Figure-(4) shows the variation of velocity v with magnetic parameter M. It is observed that the velocity v decreases as M increases. Figure-(5) shows that an increase in porosity parameter K causes an increase in velocity profile of dust particles v. From Figure-(6), it is observed that the velocity of dust particles v decreases as the mass concentration (l) of the dust particle increase. The skin friction at the plate is shown in fig. - (7) for different value of M, l, t and K. It is noted that for fixed α = 0.4, the skin friction decreases continuously with increasing time t. It is observed that the skin friction decreases with the increasing value of K, but the skin friction increases with the increasing values of M and l.

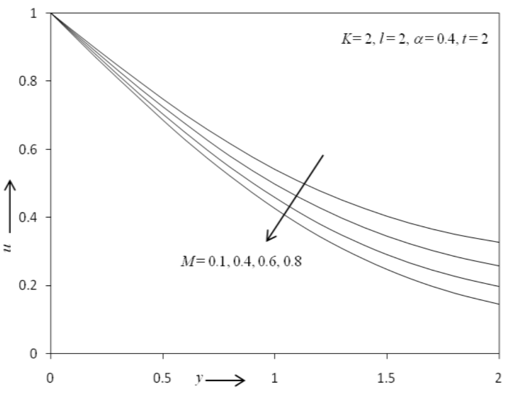

Fig. - 1: The velocity profile of fluid for different values of M.

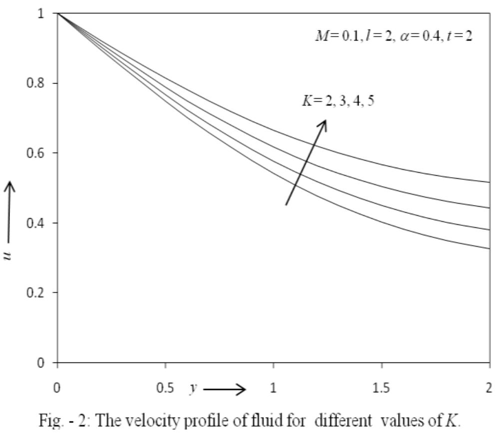

Fig. - 2: The velocity profile of fluid for different values of K.

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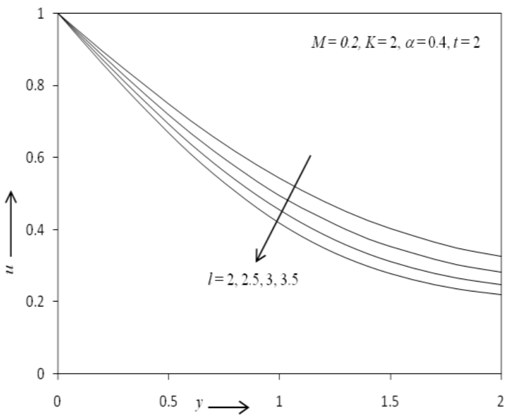

Fig. - 3: The velocity profile of fluid for different values of *l*.

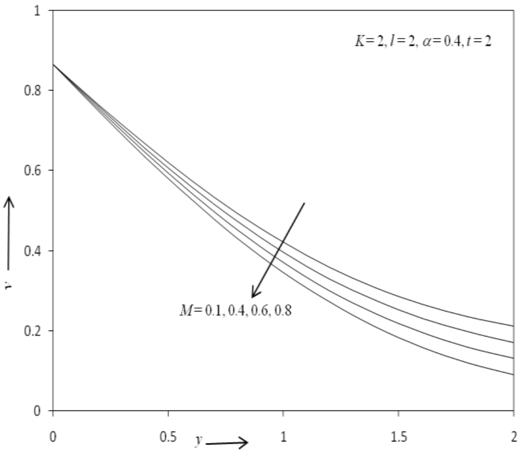

Fig. - 4: The velocity profile of dust particle for different values of *M*.

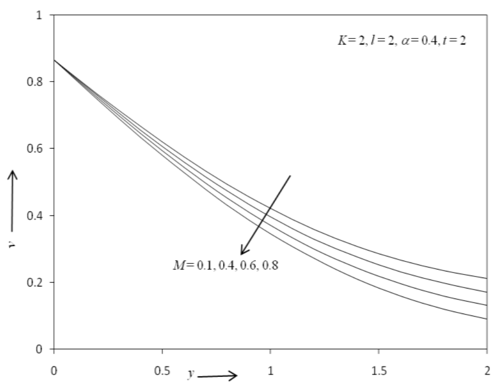

Fig. - 4: The velocity profile of dust particle for different values of *M*.

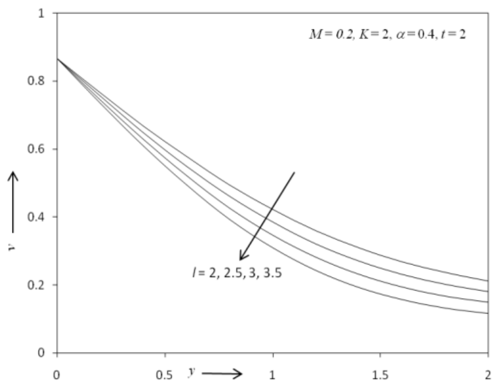

Fig. - 6: The velocity profile of dust particle for different values of *l*.

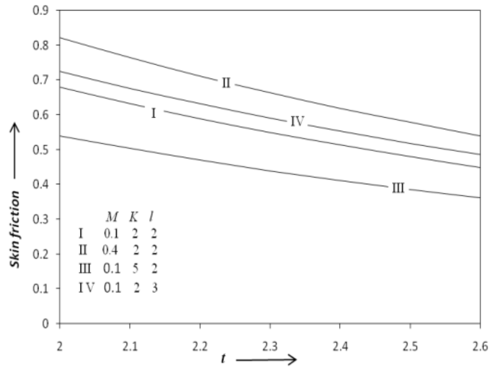

Fig.-(7), The skin friction at the plate for different value of *M*, *K* and *l*.

IV. CONCLUSION

The present note on "study of a second order (Rivlin-Ericksen) MHD fluid flow due to an impulsively started flat plate through porous medium "is concluded as: The mixed particles show the symmetric behavior to the fluid as source velocity for them is fluid velocity. Fluid velocity decreases for increasing values of M&l while increases for for increasing values of K. Velocity of dust particles decreases for increasing values of M & l while increases for increasing values of K. Skin friction increases for increasing values of M & l while decreases for increasing values of K Our results are agree with the study of Shrivastava and Khare and the fluids of second order are very much useful in industry applications relative to high drag force and high porous medium. Modal is convergent and validate for numerical method.

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