

Unsteady Magnetohydrodynamic Free Convective Flow Past a Semi-finite Vertical Porous Plate

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Abstract—In this article, we studied the effects of variable viscosity and thermal conductivity on an unsteady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid past a semi-infinite vertical plate taking into account the mass transfer. The fluid viscosity is assumed to vary as a linear function of temperature. The governing equations for the flow are transformed into a system of non-linear ordinary differential equations are solved by a perturbation technique. The effects of the various parameters on the velocity, temperature, concentration and skin-friction profiles are presented graphically and discussed qualitatively.

Index Terms—Radiation parameter, Thermal conductivity, MHD, porous medium and viscosity.

1. INTRODUCTION

Natural convection flow over vertical surfaces immersed in porous Media has paramount importance because of its notable applications in soil Physics, geohydrology, and filtration of solids from liquids chemical Engineering and biological systems. Study of fluid flow in porous medium is Based upon the empirically determined darcy's law. Such flows are Considered to be useful in diminishing the free convection, which would Otherwise occur intensely on a vertical heated surface. Recent developments In modern technology have intensified more interest of many researchers in Geothermal and oil reservoir engineering as well as other geo-physical and astrophysical studies.

Cramer, K.R and Pai, S.I. [1] taken transverse applied magnetic field And magnetic Reynolds number are assumed to be very small, such that the Induced magnetic field is negligible. Muthucumaraswamy et. Al.,[2] have Studied the effect of homogeneous chemical reaction of first order and free Convection on the oscillating infinite vertical plate with variable temperature And mass diffusion. Sharma[3] investigated the effect of periodic heat and Mass transfer on the unsteady free convection flow past a vertical flat plate in Slip flow regime when suction velocity oscillates in time. Chaudhary and Jha[4] studied the effect of chemical reaction on MHD micro polar fluid Flow past a vertical plate in slip in slip-flow regime. Anjalidevi, et al [5] have Examined the effect of chemical reaction on the flow in the presence of heat Transfer and magnetic field. Al-Odat and Al-Azab[7] studied the influence of Magnetic field on unsteady free convective heat and mass Transfer flow along an impulsively start semi-infinite vertical plate taking into account a homogeneous chemical reaction of first order.

3. MATHEMATICAL FORMULATION

Two dimensional unsteady, laminar, natural convection flows of a viscous, incompressible, electrically conducting radiating fluid past an impulsively started semi-finite vertical plate in the presence of transverse magnetic field with viscous dissipation is assumed to be gray, thermally absorbing-emitting but non-scattering. The usual Cartesian coordinate system is chosen for mathematical treatment. The X- axis is taken along the plate in the upward direction and y-axis is taken normal to it. The fluid is assumed to be slightly conducting and hence the magnetic Reynolds is much less than unity and the induced magnetic field is negligible in comparison with transverse applied magnetic field first, it is assumed that the plate and the fluid are at the same temperature and concentration level everywhere in the fluid. At time, the plate starts moving impulsive starts moving impulsive in the vertical direction with constant velocity against the gravitational field. The temperature of the plate and the concentration level of the plate are raised to and, respectively and are maintained constantly thereafter. It is assumed that the concentration of the diffusing species in the binary mixture is very less in the comparison to the other chemical species, which are present and hence the Soret and Dufour Effect are negligible. It is also assume that there is no chemical reaction between the above assumptions, in the absence of an input electronic field.

The basics governing equation for a boundary layer two-dimensional fluid flow are given by
Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

The momentum equation $\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + U \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial y^2}$

The Energy equation $\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} \right) \frac{1}{c_p}$

The Species concentration $\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = D \frac{\partial^2 c}{\partial y^2}$

Based on the assumption adopted for the present case and by the Boussinesq's approximation the above equation become.
Continuity equation

$$\frac{\partial v}{\partial y} = 0$$

Momentum equation

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

$$= g\beta(T - T_\infty) + g\beta^* (C - C_\infty) + \partial \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u - \frac{\partial}{\partial y} u$$

Energy conservation

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{\partial}{\partial y} \frac{\partial u}{\partial y}$$

Species concentration

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - K' r (C - C_\infty)$$

Here,

(u,v) – velocity components in (x,y) directions

G – acceleration due to gravity

T – temperature of the fluid

C – The species concentration

B – The co – efficient of thermal expansion

∂ – The kinematic viscosity of the fluid

K – The effective of the fluid

E – The density of the fluid

K' – The permeability

D – The diffusion co – efficient

B0 – The electromagnetic induction

Σ – The conductivity of the fluid

Cp – The specific heat at constant pressure

V0 – Constant suction / injection

The initial and boundary condition are as follows:

$$T \leq 0, u=0, v=0, T_\infty, C=C_\infty \forall y$$

$$t > 0, u = u_0, v = 0, T = T_w, C = C_w \text{ at } y = 0$$

$$u \rightarrow 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \text{ as } y \rightarrow \infty$$

Thermal radiation is assumed to be present to be present in the form of a Unidirectional flux in the y-direction i.e., qr (transverse to the vertical Surface). By using the Rosseland approximation, the radiative heat flux qr is Given by

$$q_r = -\frac{4\sigma_s}{3Kc} \frac{\partial T'^4}{\partial y}$$

where σ_s is the Stefan-Boltzmann constant and Kc the mean absorption co-efficient in the Rossel and approximation, the present analysis is limited to optically thick fluids. If temperature difference within the flow are sufficiently small, then equation (3.6) can be linearized by expanding T'^4 into the Tylor series about T'_∞ , which after neglecting higher order terms takes the form:

$$T'^4 \cong 4 T'_\infty^3 T' - 3 T'^4_\infty$$

$$\frac{\partial q_r}{\partial y} = \frac{16\sigma_s T'^3_\infty}{3kc} \frac{\partial T'}{\partial y}$$

In view of equation (3.6), (3.7) and (3.3) reduces to

$$\frac{\partial T'}{\partial t} + u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial y} = \alpha \frac{\partial^2 T'}{\partial y^2} + \frac{16\sigma_s T'^3_\infty}{3kc \rho C_p} \frac{\partial T'}{\partial y} + \frac{\partial}{\partial y} \frac{\partial u}{\partial y}$$

$$X = \frac{xu_0}{v}, y = \frac{xu_0}{v}, t = \frac{t'u_0^2}{v}, Kr = \frac{K'rv}{u_0^2},$$

$$U = \frac{u}{u_0}, v = \frac{v}{u_0}, Gr = \frac{vg\beta(T'_w - T'_\infty)}{u_0^3}$$

$$Gm = \frac{vg\beta^*(C'_w - C'_\infty)}{u_0^3}, N = \frac{KcK}{4\sigma_s T'^3_\infty}, M = \frac{\sigma\beta^2}{u_0},$$

$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, K = \frac{K'u_0^3}{v^2}$$

$$Pr = \frac{\theta}{\alpha}, Sc = \frac{\theta}{\alpha}$$

Now, where only one dimensional flow is considered, the above set of equation (3.1), (3.2), (3.8), and (3.4) are reduce to the following non-dimensional form:

$$\frac{\partial v}{\partial y} = 0 \rightarrow v = -v_0 \text{ (where } v_0 = 1)$$

$$\frac{\partial u}{\partial t} \frac{\partial u}{\partial y} = Gr T + Gm C + \frac{\partial^2 T}{\partial y^2} - M + \frac{1}{K} U$$

$$\frac{\partial T}{\partial t} \frac{\partial T}{\partial y} = \left(\frac{1}{Pr} + 1 + \frac{4}{3N} \right) \frac{\partial^2 T}{\partial y^2}$$

$$\frac{\partial C}{\partial t} \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - Kr C$$

Here,

Gr - Grashoff number

Gm - Modified Grashof number

M - Magnetic number

Sc - Schmidt number

K - permeability parameter porous medium

Pr - prandtl number

ω - the frequencies of oscillation

F - the non- dimensional velocity

U - the non – dimensional velocity

The corresponding initial and boundary condition are as follows

$$\left. \begin{array}{l} t \leq 0, U = 0, T = 0, C = 0 \quad \forall y \\ t > 0, U = 1, T = 1, C = 1 \text{ at } y=0 \\ u \rightarrow 0, T \rightarrow 0, C \rightarrow 0 \text{ as } y \rightarrow \infty \end{array} \right\}$$

4. SOLUTION OF THE PROBLEM

These equation (3.11) – (3.13) are coupled and hence to be solved simultaneously. The closed form solution is difficult and hence Approximation method can be used. One among them is the perturbation Method. As ω is so small that it may be taken as the perturbation parameter in order to solve the equations (3.11) (3.12) and (3.13) with respect to the Boundary condition (3.14) for the flow let us take

$$U(y,t) = U_0(y) + U_1(1) e^{\omega t}$$

$$T(y,t) = T_0(y) + T_1(1) e^{\omega t}$$

$$C(y,t) = C_0(y) + C_1(1) e^{\omega t}$$

Substituting the expression (3.15) (3.16) and (3.17) in (3.11)(3.12) and (3.13), we get

$$\begin{aligned} & \frac{\partial(U_0 - U_1 e^{\omega t})}{\partial t} - \frac{\partial(U_0 - U_1 e^{\omega t})}{\partial y} \\ & = Gr (T_0 + T_1 e^{\omega t}) + Gm(C_0 + C_1 e^{\omega t}) + \frac{\partial^2 (U_0 - U_1 e^{\omega t})}{\partial y^2} - M + \frac{1}{K} (U_0 - U_1 e^{\omega t}) \end{aligned}$$

Equation the terms not containing exponential and the co-efficient of $e^{\omega t}$ we have

$$U''_0 + U'_0 - (M + \frac{1}{K}) U_0 = -(Gr T_0 + Gm C_0)$$

$$U''_1 + U'_1 - (M + \frac{1}{K} + \omega) U_1 = -(Gr T_1 + Gm C_1)$$

Now, using (3.16) in (3.12) we get

$$\frac{\partial(T_0 - T_1 e^{\omega t})}{\partial t} - \frac{\partial(T_0 - T_1 e^{\omega t})}{\partial y} = \frac{1}{Pr} - M + \frac{1}{K} + \frac{\partial^2 (T_0 - T_1 e^{\omega t})}{\partial y^2}$$

Equating the term not containing exponential and the co- efficient of $e^{\omega t}$ we have

$$T_0'' + K_2 T_0' = 0 \quad T_1'' + K_2 T_1' + \omega T_1 = 0$$

$$\text{Now, using (3.17) in (3.13) we get } \frac{\partial(C_0 - C_1 e^{\omega t})}{\partial t} - \frac{\partial(C_0 - C_1 e^{\omega t})}{\partial y} = \frac{1}{Sc} \frac{\partial^2 (C_0 - C_1 e^{\omega t})}{\partial y^2} - Kr(C_0 + C_1 e^{\omega t})$$

Equation the term not containing exponential and the co-efficient of $e^{\omega t}$

$$C_1'' + Sc C_0' - Sc Kr C_0 = 0 \quad C_1'' + Sc C_0' - (Kr + \omega) C_1 = 0$$

Where prime denotes ordinary differentiation with respect to y. The corresponding boundary conditions can be written as

$$U_0 = 1, T_0 = 1, T_1 = 1, C_0 = 1, C_1 = 0 \text{ at } y=0, \quad U_0 = 0, T_0 = T_1, C_0 = 1, C_1 = 0 \text{ at } y \rightarrow \infty$$

By equation (3.22)
 $C_0'' + Sc \ C_0' - Sc \ Kr \ C_0 = 0$
 $C_0(y) = a_1 e^{-m_1 y} + a_2 e^{-m_2 y}$
 Applying the boundary conditions(3.24)
 $C_0(y) = e^{-m_1 y}$
 By equation (3.20), $T_0'' + k_{12} \ T_0' = 0$, $T_0(y) = b_1 + b_2 e^{-m_2 y}$
 Applying the boundary conditions (3.24), $T_0(y) = e^{-m_2 y}$
 By equation (3.18), $U_0'' + U_0' - (M + \frac{1}{K}) \ U_0 = -(Gr \ T_0 + Gm \ C_0)$
 $U_0'' + U_0' - (M + \frac{1}{K}) \ U_0 = -(Gr e^{-m_2 y} + Gm e^{-m_1 y})$

$U_0(y) = -A_1 e^{-m_1 y} - A_2 e^{-m_2 y}$
 Applying the boundary condition(3.24), $U_0(y) = (1+A_3)e^{-m_3 y} - A_1 e^{-m_2 y} - A_2 e^{-m_1 y}$
 We obtain the velocity temperature and concentration distribution in the Boundary layer as
 $U_0(y,t) = (1+A_3)e^{-m_3 y} - A_1 e^{-m_2 y} - A_2 e^{-m_1 y}$
 $T(y,t) = e^{-m_2 y}$, $C(y,t) = e^{-m_1 y}$
 Where the expression for constant are given in the Appendix.

4.1 SKIN –FRICTION

The dimensionless shearing stress on the surface of a body, due to the fluid motion, is known as skin as skin-friction and is defined by the Newton's law of viscosity.

The Skin – friction is

$$\tau = \left(\frac{\partial u}{\partial y} \right)_{y=0} = \frac{\partial (1+A_3 e^{-m_3 y} - A_1 e^{-m_2 y} - A_2 e^{-m_1 y})}{\partial y}$$

$$\tau = -m_3(1+A_3) + A_1 m_2 + A_2 m_1$$

Where the expressions for the constant are given in the Appendix.

5. RESULTS AND DISCUSSION

In order to get a physical insight in to the problem the effects of various governing parameters on the physical quantities are computed and represented in Figures 1-13 and discussed in detail. The formulation of the effects of chemical reaction and heat absorption on MHD convective flow and mass transfer of an incompressible, viscous fluid along a semi infinite vertical porous moving plate in a porous medium has been performed in the preceding sections. This enables us to carry out the numerical calculations for the distribution of the velocity, temperature and concentration across the boundary layer for various values of the parameters.

The influence of Magnetic field on the velocity profiles has been studied in Fig .1. It is seen that the increase in the applied magnetic intensity contributes to the decrease in the velocity. Further, it is seen that the magnetic influence does not contribute significantly as we move away from the bounding surface. The influence of the porosity of the boundary on the velocity of the fluid medium has been shown in Fig 2. It is seen that as the porosity of the fluid bed increases, the velocity also increases which is in tune with the realistic situation.

Fig.10 illustrates the influence of the radiation parameter on the temperature profiles in the boundary layer. As radiation parameter increases, temperature distributions increase when the other physical parameters are fixed.

The influence of Schmidt number on the concentration is illustrated in Fig. 11. It is observed that increase in Sc contributes to decrease of concentration of the fluid medium. Further, it is seen that Sc does not contributes much to the concentration field as we move far away from the bounding surface. Fig.12 shows that the effect of increasing the chemical reaction parameter on concentration profiles. It is noticed that species concentration are decreasing, as the values of chemical reaction are increasing. The effect of chemical reaction on velocity and temperature is less dominant in comparison to concentration. Skin friction for various values of magnetic field strength is portrayed through Fig 13. It is seen that skin friction decreases, as magnetic parameter increases, whereas it is increasing when radiation parameter is increasing.

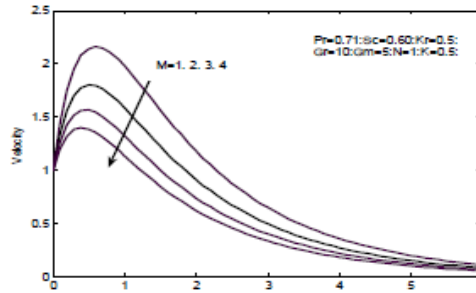


Fig.1. Effects of magnetic parameter on velocity profiles.

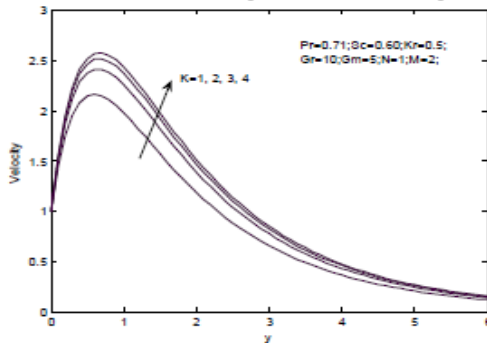


Fig.2. Effects of permeability parameter on velocity profiles

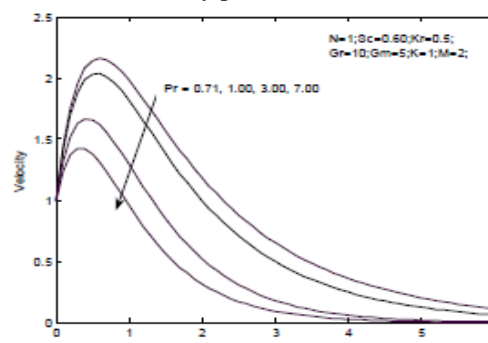


Fig.3. Effects of radiation parameter on velocity profiles

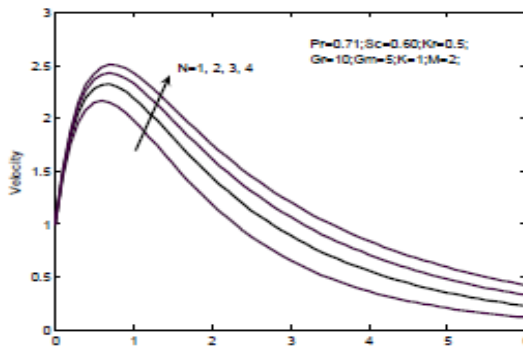


Fig.4. Effects of Prandtl number on velocity profiles.

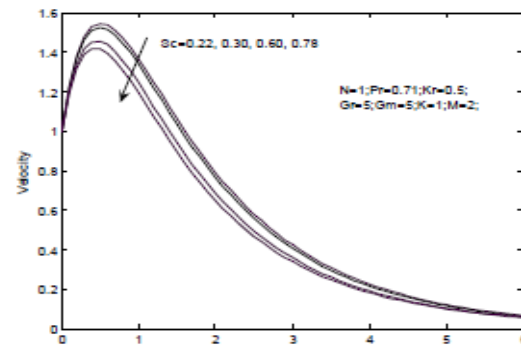


Fig.5. Effects of Schmidt number on velocity profiles.

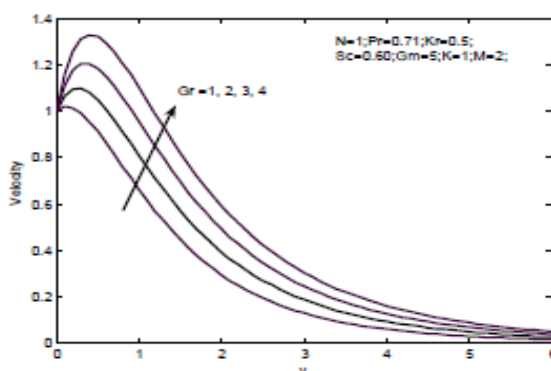


Fig.6. Effects of Grashof number on velocity profiles.

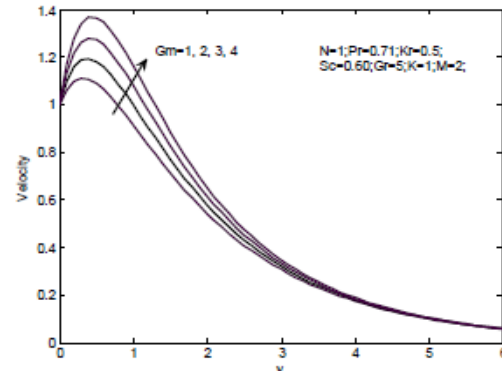


Fig.7. Effects of modified Grashof number on velocity profiles.

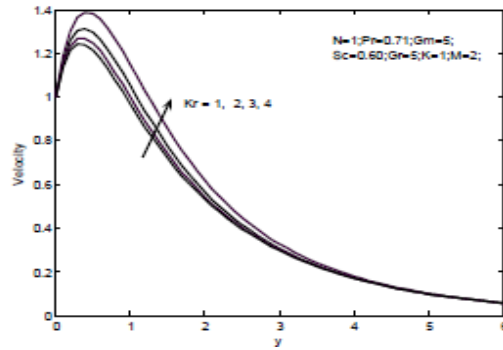


Fig.8. Effects of magnetic parameter on velocity profiles.

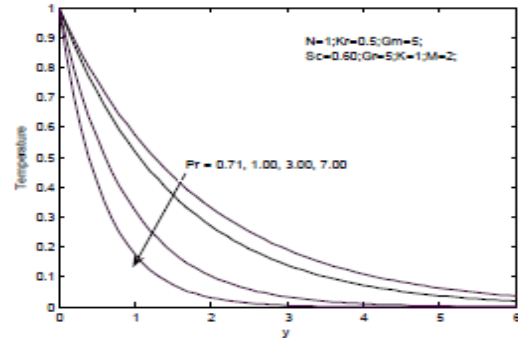


Fig.9. Effects of Prandtl number on temperature profiles.

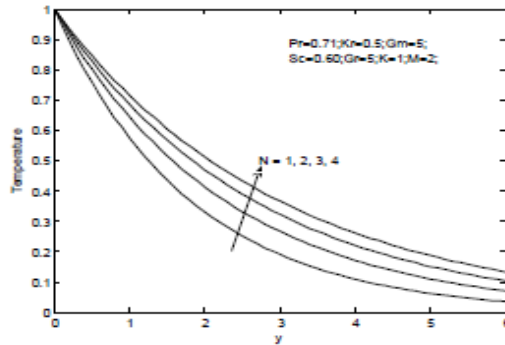


Fig.10. Effects of radiation parameter on temperature profiles.

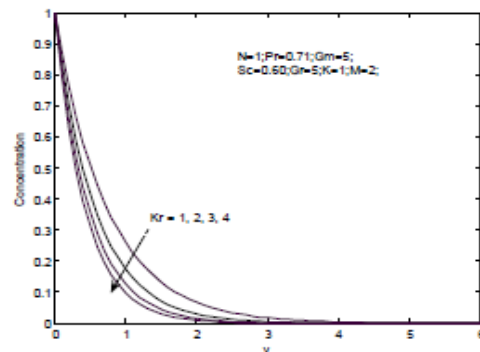


Fig.11. Effects of chemical reaction parameter on concentration profiles

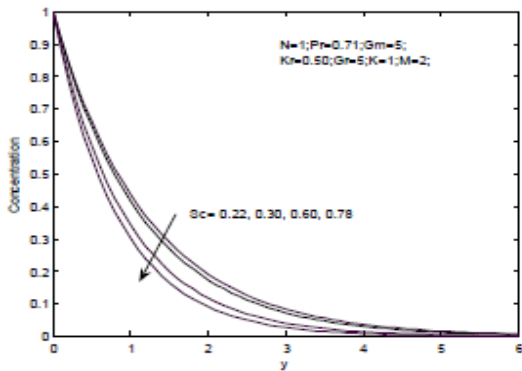


Fig.12. Effects of Schmidt number on concentration profiles.

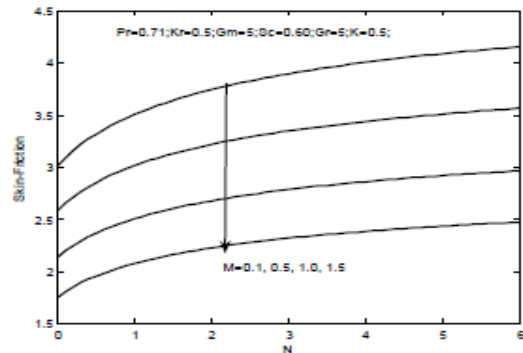


Fig.13. Effects of magnetic parameter on skin-friction.

6. CONCLUSION

In this article a mathematical model has been presented for the thermal radiation effect on unsteady magneto hydrodynamic flow past a vertical porous plate with variable suction. The non-dimensional governing equations are solved with the help of finite element method. The conclusions of the study are as follows:

- The velocity increases with the increase Grashof number and modified Grashof number.
- The velocity decreases with an increase in the magnetic parameter.
- The velocity increases with an increase in the permeability of the porous medium parameter.
- Increasing the Prandtl number substantially decreases the velocity and the temperature function.
- The velocity as well as concentration decreases with an increase in the Schmidt number.

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