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A Derivation to Overall Murphree Efficiency from Individual Efficiencies for a Special Case in Staged Operations

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Abstract: *In order to improve the stage operation such as extraction with a minimum of plant alteration, the effluents from the stage will both be led to another sage identical in construction. The combination, two stages in series, is in effect a single stage. The combination, two stages or more in series, is in effect a single stage is a simple technique to improve the existing operation. In this work, we have presented a derivation that relates combined stage overall Murphree efficiency to individual stage efficiency.*

Keywords *- Staged Operation; Series Operation; Murphree Stage Efficiency; Overall Murphree Stage Efficiency*

I. INTRODUCTION

In order to improve the performance of the staged operations with a minimum of plant alteration, the effluents from the stage will both be led to another sage identical in construction the combination, two stages in series, is in effect a single stage [1]. In this manner any numbers of stages have been recommended to connect in series. Ultimately we bring the flowing streams close to equilibrium [2]. The efficiencies of individual stage may be reported as Murphree stage efficiencies. This efficiency is defined as [3,4] the actual vapour stream enrichment over one stage divide by the theoretical vapour stream enrichment which would have been obtained if the liquid stream on the stage and vapours stream leaving the plate had reached equilibrium. Thus, if a vapour stream of composition y_n enters stage $n-1$, the actual vapours evolved have a composition of y_{n-1}. If the evolved vapours had left the liquid on the stage $n-1$ under equilibrium conditions, the composition of the vapours stream would have been y^*_{n-1}. Therefore, the actual vapour stream enrichment is then $y_{n-1} - y_n$ and the theoretically vapour stream enrichment is $y^*_{n-1} - y_n$ therefore

Murphree stage efficiency for vapour stream for stage, $n-1$, $\eta_{ME} = \dfrac{y_{n-1} - y_n}{y^*_{n-1} - y_n}$

There is another definition for stage efficiency which is defined as the ratio of actual enrichment to possible enrichment in a single stage. Therefore

Stage efficiency for vapour stream for stage, $n-1$, $\eta = \dfrac{y_{n-1} - y_n}{y^*_e - y_n}$

In the following section a theoretical derivation has been derived to compute the overall Murphree stage efficiency, η_{MEO} from the individual efficiencies.

II. DERIVATION

Solute free (i.e., mole ratios) coordinates are very useful in staged operation [1]. Mole ratios are usually designated by Y and X, whereas mole fractions are designated by y and x for vapour stream and liquid stream respectively. However, mole ratios are equal to mole fractions for very dilute systems. Hence mole ratios are used in the following derivation.

Referring to Figure-I, when solute is transferred in a single stage. The following definitions are applicable.

Murphree stage efficiency (η_{ME}) is defined as $\frac{Y_2 - Y_1}{Y_2^* - Y_1}$;Efficiency (η) is defined as $\frac{Y_2 - Y_1}{Y_e - Y_1}$

Where subscripts 1 and 2 indicate inlet and outlet of a stage

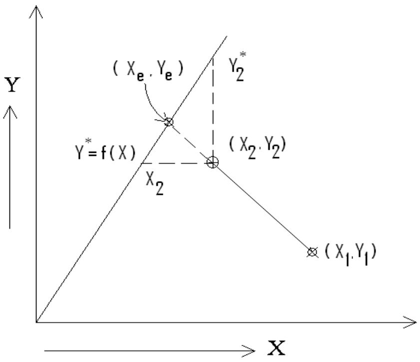

Figure-I. Single stage transfer

Referring to Figure-II when solute is transferred in multistage.

For stage one efficiency, $\eta_1 = \frac{Y_2 - Y_1}{Y_e - Y_1}$,

For stage two efficiency, $\eta_2 = \frac{Y_3 - Y_2}{Y_e - Y_2}$ and so on for nth stage, $\eta_n = \frac{Y_{n=1} - Y_n}{Y_e - Y_n}$

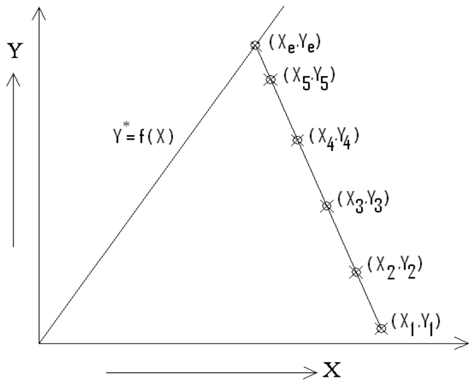

Figure-II. Multistage transfer

Let several sages are combined in series as shown in Figure-II and assuming all are having equal efficiencies

When two stages combined that is

$$\therefore \eta(Y_e - Y_1) = Y_2 - Y_1 \quad (1)$$

$$\eta(Y_e - Y_2) = Y_3 - Y_2 \quad (2)$$

Add Eq. (1) & Eq. (2) gives

$$Y_3 - Y_2 = \eta[Y_e - Y_1 + Y_e - Y_2] \quad (3)$$

Divide both sides by $(Y_e - Y_1)$

$$\therefore \quad \frac{Y_3 - Y_1}{Y_e - Y_1} = \eta\left[1 + \frac{Y_e - Y_2}{Y_e - Y_1}\right] \quad (4)$$

We know $\quad \eta_2 = \frac{Y_3 - Y_1}{Y_e - Y_1} \quad (5)$

therefore $\quad \eta_{n=2} = \eta\left[1 + \frac{Y_1 - Y_1 + Y_e - Y_2}{Y_e - Y_1}\right] \quad (6)$

implies $\quad n_{n=2} = \eta[1 + (1-\eta)] \quad (7)$

Let three stages are combined

Similarly $\quad \frac{Y_4 - Y_1}{Y_e - Y_1} = \eta\left[1 + \frac{Y_e - Y_1 + Y_e - Y_2 + Y_e - Y_3}{Y_e - Y_1}\right] \quad (8)$

We know $\quad \eta_{n=3} = \frac{Y_4 - Y_1}{Y_e - Y_1} \quad (9)$

$$\therefore \quad \eta_{n=3} = \eta\left[1 + \frac{Y_e - Y_2}{Y_e - Y_1} + \frac{Y_e - Y_3}{Y_e - Y_1}\right] \quad (10)$$

$$\eta_{n=3} = \eta\left[1 + \frac{Y_e - Y_1 + Y_1 - Y_2}{Y_e - Y_1} + \frac{Y_e - Y_1 + Y_1 - Y_3}{Y_e - Y_1}\right] \quad (11)$$

$$= \eta[1 + (1-\eta) + 1 - \eta_{n-2}]$$

$$= \eta\left|1 + (1-\eta) + 1 + \eta^2 - 2\eta\right|$$

$$= \eta\left|1 + (1-\eta) + (1-\eta)^2\right| \quad (12)$$

So on when ‘n’ stage are combined

$$\therefore \quad \eta_{n=n} = \eta\left|1 + (1-\eta) + (1-\eta)^2 + - - - - - - - +(1-\eta)^{n-1}\right| \quad (13)$$

Where $\eta_{n=n}$ is called as overall efficiency when ‘n’ stages are combined.

III. **Relation between η_{ME} and η**

It is possible to derive a relation between η_{ME} and η only when we establish a relation between them and it is possible only in a special case such as when equilibrium line and operating lines are straight lines.

Slope of the equilibrium line, $m = \frac{Y_e - Y_2}{X_e - X_2^*} \text{(or)} \frac{Y_2^* - Y_e}{X_2 - X_e} \text{(or)} \frac{Y_2^* - Y_2}{X_2 - X_2^*}$ (14)

Slope of operating line, $\frac{R_s}{E_s} = \frac{Y_2 - Y_1}{X_1 - X_2} \text{(or)} \frac{Y_e - Y_1}{X_1 - X_e} \text{(or)} \frac{Y_e - Y_2}{X_2 - X_e}$ (15)

We know $\eta_{ME} = \frac{Y_2 - Y_1}{Y_2^* - Y_1}$ and $\eta = \frac{Y_2 - Y_1}{Y_e - Y_1} = \frac{X_2 - X_1}{X_e - X_1}$ (16)

Therefore, η_{ME} can be arranged in terms of η as follows

$$\eta_{ME} = \frac{\eta(Y_e - Y_1)}{Y_2^* - Y_1} = \frac{\eta}{\frac{Y_2^* - Y_1}{Y_e - Y_1}} = \frac{\eta}{\frac{Y_2^* - Y_1}{Y_e - Y_1} - 1 + 1} = \frac{\eta}{\frac{Y_2^* - Y_e}{Y_e - Y_1} + 1} \quad (17)$$

$$\eta_{ME} = \frac{\eta}{\frac{m(X_2 - X_e)}{(Y_e - Y_1)} + 1} = \frac{\eta}{\frac{m(X_2 - X_e)}{\frac{R_s}{E_s}(X_1 - X_e)} + 1} = \frac{\eta}{\frac{mE_s}{R_s}\left(\frac{X_2 - X_e}{X_1 - X_e}\right) + 1} \quad (18)$$

We know $\eta = \frac{X_2 - X_1}{X_e - X_1}$ therefore $\eta - 1 = \frac{X_2 - X_e}{X_e - X_1}$ (19)

$$\therefore \eta_{ME} = \frac{\eta}{\frac{mE_s}{R_s}(1 - \eta) + 1} \quad (20)$$

$S = \frac{mE_s}{R_s}$, stripping factor

$$\therefore \eta_{ME} = \frac{\eta}{S(1 - \eta) + 1} \quad (21)$$

Rearranging Eq.(21)

$$\eta = \frac{(S + 1)\eta_{ME}}{S\eta_{ME} + 1} \quad (22)$$

The following examples illustrate how to use Eq.(21) and Eq.(22) in computing overall Murphee stage efficiency from individual Murphree stage efficiency

Example: A single stage liquid-extraction operation with a linear equilibrium distribution curve (Y = m X at equilibrium) operates with a stripping factor S = m E_s/R_s = 1.0 and has a Murphree stage efficiency $\eta_{ME} = 0.6$. In order to improve the extraction with a minimum of plant alteration, the effluents from the stage will both be led to another sage identical in construction, also of efficiency $\eta_{ME} = 0.6$. The combination, two stages in series, is in effect a single stage. It is desired to compute the overall Murphree stage efficiency η_{MEO} of the combination [2].

Solution:

Given S = 1.0 and $\eta_{ME} = 0.6$

Therefore $\eta = \frac{(S+1)\eta_{ME}}{S\eta_{ME}+1} = 0.75$

$$\eta_{n=2} = \eta\left[1+(1-\eta)\right] = 0.9375$$

$$\eta_{MEO} = \frac{\eta_{n=2}}{S(1-\eta_{n=2})+1} = 0.8824$$

3. CONCLUSION

A theoretical derivation has been presented to compute overall Murphree stage efficiency from individual stage efficiency. We have presented a model problem along with the calculations to calculate the overall efficiency from individual efficiency. Although the derivations for other cases are not present here, the same concept can be extended to absorption, stripping and adsorption.

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