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Stability of Duodecic Functional Equations in Multi-Banach Spaces

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Abstract - In this paper, we carry out the Hyers- Ulam Stability of duodecic functional equation equation in Multi-Banach Spaces by using fixed point method.

I. INTRODUCTION

In 1940, Ulam posed a problem concerning the stability of functional equations: Give conditions in order for a linear function near an approximately linear function to exist. An earlier work was done by Hyers [6] in order to answer Ulam's equation [13] on approximately additive mappings.During last decades various stability problems for large variety of functional equations have been investigated by several mathematicians. A large list of references concerning in the stability of functional equations can be found. e.g.([1], [2], [5], [6], [7], [8], [9],[14]).

In this paper, we carry out the Hyers-Ulam stability of duodecic functional equation

$G(x;y) = f(x+6y) - 12f(x+5y) + 66f(x+4y) - 220f(x+3y) + 495f(x+2y) - 792f(x+y) + 924f(x) - 792f(x-y) + 495f(x-2y) - 220f(x-3y) + 66f(x-4y) - 12f(x-5y) + f(x-6y) - 12!\,f(y)$

where $12! = 479001600$, in Multi-Banach Spaces by using fixed point approach. It is easily verified that that the function $f(x) = x^{12}$ satisfies the above functional equations. In other words, every solution of the duodecic functional equation is called a dodecic mapping. Now, we present the following theorem due to B. margolis and J.B. Diaz [3], [11] for the fixed point theory.

Theorem 1 [3], [11] Let (x,d) be a complete generalized metric space and let $J: X \to X$ be a strictly contractive mapping with Lipschitz constant $L < 1$. Then for each given element $x \in X$, either $d(J^n x, J^{n+1} x) = \infty$

for all nonnegative integers n or there exists a positive integer n_0 such that

i. $d(J^n x, J^{n+1} x) < \infty$ for all $n \geq n_0$;
ii. (ii) The sequence $\{J^n x\}$ is convergent to a fixed point y^* of J ;
iii. y^* is the unique fixed point of T in the set $Y = \{y \in X: d(J^{n_0} x, y) < \infty\}$;
iv. $d(y, y^*) \leq \frac{1}{1-L}\, d(y, Jy)$ for all $y \in Y$.

Theorem 2

Let A be an linear space and let $\left((B^k, \|.\|_k): k \in \mathbb{N}\right)$ be a multi-Banach space. Suppose that η is a non-negative real number and $f: A \to B$ is a mapping fulfills

$sup_{k\in\mathbb{N}}\left\|\left(Gf(x_1,y_1),\ldots,Gf(x_k,y_k)\right)\right\|_k \leq \eta$ ……………………..(1)

For all $x_1, x_2, \ldots x_k, y_1, \ldots y_k \in A$. Then there exists a unique duodecic mapping $D: A \to B$ such that

$sup_{k\in\mathbb{N}}\left\|\left(f(x_1) - D(x_1),\ldots,f(x_k) - D(x_k)\right)\right\|_k \leq \frac{16781312}{8034351319000000}\,\eta$ ………..(2)

For all $x_1, x_2, \ldots x_k \in A$.

Proof.

Doing $x_1 = x_2 =,\ldots,= x_k = 0$ and changing $y_1,\ldots,y_k$ by $2x_1,\ldots,2x_k$ in (1), and dividing by 2 in the resulting equation, we get

$sup_{k\in\mathbb{N}}\|(f(12x_1) - 12f(10x_1) + 66f(8x_1) - 220f(6x_1) + 495f(4x_1) - 239501592f(2x_1),\ldots,f(12x_k) - 12f(10x_k) + 66f(8x_k) - 220f(6x_k) + 495f(4x_k) - 239501592f(2x_k))\| \leq \frac{\eta}{2}$ ……………………………………………………...(3)

forall $x_1,\ldots,x_k \in A$. Taking $x_1,\ldots,x_k$ by $6x_1,\ldots,6x_k$ and changing $y_1,y_2,\ldots,y_k$ by $x_1,\ldots,x_k$ in (1), we have

$sup_{k\in\mathbb{N}}\|(f(12x_1) - 12f(11x_1) + 66f(10x_1) - 220f(9x_1) + 495f(8x_1) - 792f(7x_1) + 924f(6x_1) - 792f(5x_1) + 495f(4x_1) - 220f(3x_1) + 66f(2x_1) - 479001612f(x_1),\ldots,f(12x_k) - 12f(11x_k) + 66f(10x_k) - 220f(9x_k) + 495f(8x_k) - 792f(7x_k) + 924f(6x_k) - 792f(5x_k) + 495f(4x_k) - 220f(3x_k) + 66f(2x_k) - 479001612f(x_k))\| \leq \eta$ ……………………………………………………..(4)

forall $x_1,\ldots,x_k \in A$. Combining (3) and (4), one gets

$sup_{k\in\mathbb{N}}\|12f(11x_1) - 78f(10x_1) + 220f(9x_1) - 429f(8x_1) + 792f(7x_1) - 1144f(6x_1) + 792f(5x_1) + 220f(3x_1) - 239501658f(2x_1) + 479001612f(x_1),\ldots,12f(11x_k) - 78f(10x_k) + 220f(9x_k) - 429f(8x_k) + 792f(7x_k) - 1144f(6x_k) + 792f(5x_k) + 220f(3x_k) - 239501658f(2x_k) + 479001612f(x_k)\|_k \leq \frac{3}{2}$ ………………….(5)

forall $x_1,\ldots,x_k \in A$. Taking $x_1,\ldots,x_k$ by $5x_1,\ldots,5x_k$ and changing $y_1,y_2,\ldots,y_k$ by $x_1,\ldots,x_k$ in (1), we have

$sup_{k\in\mathbb{N}}\|f(11x_1) - 12f(10x_1) + 66f(9x_1) - 220f(8x_1) + 495f(7x_1) - 792f(6x_1) + 924f(5x_1) - 792f(4x_1) + 495f(3x_1) - 220f(2x_1) + 479001533f(x_1),\ldots,f(11x_k) - 12f(10x_k) + 66f(9x_k) - 220f(8x_k) + 495f(7x_k) -$

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$792f(6x_k) + 924f(5x_k) - 792f(4x_k) + 495f(3x_k) - 220f(2x_k) + 479001533f(x_k)\|_k \le \eta$ ………………(6)

forall $x_1,..,x_k \in A$. Multiplying by 12 on both sides of (6), then it follows from (5) and the resulting equation, we arrive at

$sup_{k\in\mathbb{N}}\|(66f(10x_1) - 572f(9x_1) + 2211f(8x_1) - 5148f(7x_1) + 8360f(6x_1) - 10296f(5x_1) + 9504f(4x_1) - 5720f(3x_1) - 239499018f(2x_1) + 6227020008f(x_1), ..., 66f(10x_k) - 572f(9x_k) + 2211f(8x_k) - 5148f(7x_k) + 8360f(6x_k) - 10296f(5x_k) + 9504f(4x_k) - 5720f(3x_k) - 239499018f(2x_k) + 6227020008f(x_k))\| \le 12\,\eta$ …………………..(7)

On Simplification we arrive at,

$sup_{k\in\mathbb{N}}\left\|\left(\frac{1}{4096}f(2x_1) - f(x_1), \dots, \frac{1}{4096}f(2x_k) - f(x_k)\right)\right\| \le \frac{4097}{1961990554000}\,\eta$ ………..(8)

forall $x_1,..,x_k \in A$.

Let $\Lambda = \{A \to B \mid l(0) = 0\}$ and introduce the generalized metric d defined on λ

$$d(l,m) = inf\{\lambda \in [0,\infty] \mid sup_{k\in\mathbb{N}}\|l(x_1) - m(x_1), \dots, l(x_k) - m(x_k)\|_k \le \lambda\ \forall x_1, \dots x_k \in A\}$$

Then it is easy to show that Λ, d is a generalized complete metric space, see[10].

We define an operator $Jl(x) = \frac{1}{2^{12}} l(2x)\ \ \forall x \in A$

We assert that J is a strictly contractive operator. Given $l, m \in \Lambda$, let $\lambda \in [0,\infty]$ be an arbitrary constant with $d(l,m) \le \lambda$. From the definition if follows that

$$sup_{k\in\mathbb{N}}\|(Jl(x_1) - Jm(x_1), \dots, Jl(x_k) - Jm(x_k))\|_k \le \frac{1}{2^{12}}\,\lambda$$

forall $x_1,..,x_k \in A$.

Hence, it holds that

$d(Jl,Jm) \le \frac{1}{2^{12}}\,\lambda d(Jl,Jm) \le \frac{1}{2^{12}}\,d(l,m) \qquad \forall l,m \in \Lambda.$

This means that J is strictly contractive operator on Λ with the Lipschitz constant $L = \frac{1}{2^{12}}$.

By(8), we have $d(Jf,f) \le \frac{4097}{1961990554000}\,\eta$. According to Theorem 1, we deduce the existence of a fixed point J that is the existence of mapping $D: A \to B$ such that

$D(2x) = 2^{12}D(x)\ \ \forall x \in A.$

Moreover, we have $d(J^n f, D) \to 0$, which implies

$D(x) = \lim_{n\to\infty} J^n f(x) = \lim_{n\to\infty}\frac{f(2^n x)}{2^{12n}}$ for all $x \in A$.

Also, $d(f,D) \le \frac{1}{1-L}\,d(Jf,f)$ implies the inequality

$d(f,D) \le \frac{16781312}{8034351319000000}\,\eta.$

Setting $x_1 =, \dots, = x_k = 2^n x,\ y_1 =,..,y_k = 2^n y$ in (1) and divide both sides by 2^{12n}. Then, using property (a) multi-norms [4], we obtain

$$\|GDf(x,y)\| \le \lim_{n\to\infty}\frac{1}{2^{12n}} = 0$$

For all $x, y \in A$. Hence D is Duodecic.

The Uniqueness of D follows from the fact that D is the unique fixed point of J with the property that there exists $\ell \in (0,\infty)$ such that

$$sup_{k\in\mathbb{N}}\|(f(x_1) - D(x_1), \dots, f(x_k) - D(x_k))\|_k \le \ell$$

forall $x_1,..,x_k \in A$.

This completes the proof of the Theorem.

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