International Journal of Computing Algorithm
Volume: 04 Issue: 02 December 2015 Pages:76-78
ISSN: 2278-2397

Solving Assignment Problem Using L-R Fuzzy Numbers

S.JohnsonSavarimuthu[1], K.Babu[2]
[1]Associate Professor, Department of Mathematics, St.Joseph's college of Arts and Science Cuddalore, India.
[2]Department of Mathematics, St.Joseph's college of Arts and Science Cuddalore, India.
Email: babugeorge43@gmail.com

Abstract-In this paper we determine a new method to solving assignment problem using L-R fuzzy parameters. This method requires finding out the minimal cost reach optimality compared to the existing methods available in the literature. Numerical examples show that the fuzzy assignment ranking method offers an effective way for handling the fuzzy assignment problem.

Keywords-Assignment problem, fuzzy quantifier, L-R fuzzy number, optimal solution, λ-cut

I. INTRODUCTION

In this work we investigate a more assignment problem namelyrealastic the assignment problem with fuzzy costs or time since the objective values of the function by ranking method. A method named YAGER'S method is proposed to find fuzzy optimal solution of fuzzy assignment problems by representing all parameters as fuzzy quantifiers which are replaced by L-R fuzzy numbers. The advantages of the proposed method solved and the results obtained are also discussed the proposed method is easy to understand and to apply for finding an optimal solution of assignment problem in real life situation or a vagueness in everyday life.

II. DEFINITION

A fuzzy set is characterized by a membership function mapping elements of a domain X→[0,1] is a mapping called the degree of membership value of x∈X in the fuzzy set A. Those membership grades are often represented by real numbers ranging from [0,1].

A. Triangular fuzzy number

Triangular fuzzy number is defined on A =(a, b, c) are real numbers and its member function is given below

$$\mu_{A(x)} = \begin{cases} \frac{x-a}{b-a}, & \text{for } a \le x \le b \\ 1, & \text{for } x = b \\ \frac{c-x}{c-b}, & \text{for } b \le x \le c \\ 0, & \text{otherwise} \end{cases}$$

III. DEFINITION

A fuzzy number $\bar{A}=(m,n,\alpha,\beta)_{L\text{-}R}$ is said to be an L-R fuzzy number if . Where L and R are continuous, non-increasing functions that define the left and right shapes of $\mu_{A(x)}$ respectively and L(0)=R(0)=1.

$$\mu_{A(x)} = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & \text{for } x \le m,\ \alpha > 0 \\ R\left(\frac{\alpha-x}{c-b}\right), & \text{for } x \ge n,\ \beta > 0 \\ 1, & \text{otherwise} \end{cases}$$

A. Definition

Let $\bar{A}=(m,n,\alpha,\beta)_{L\text{-}R}$ be an L-R fuzzy number. Then the crisp set $A_\alpha = \{x\in X:\mu_{\tilde{A}}(x)\ge\lambda\}$ and λ be a real number in the interval [0,1] then $[m-\alpha L^{-1}(\lambda)\ ,\ n+\beta R^{-1}(\lambda)]$ is said to be λ-cut of $\bar{A}$.

B. Yager's ranking approach

A number of ranking approaches have been proposed for comparing fuzzy numbers. In this paper, Yager's ranking approach is used for fuzzy numbers. This approach involves relatively simple computational and is easily understandable. This approach involves a procedure for ordering fuzzy numbers in which a ranking index $R(\tilde{A})$is calculated for an LR flat fuzzy number $\tilde{A} =(m,n,\alpha,\beta)_{LR}$ from its λ-cut $A_\lambda= [m-\alpha L^{-1}(\lambda), n+\beta R^{-1}(\lambda)]$ according to the following formula.

$$R(\tilde{A})=\frac{1}{2}\left(\int_0^1 m-\alpha L^{-1}(\lambda)d\lambda\right) +\int_0^1 (n+\beta R^{-1}(\lambda))d\lambda)$$

Let $\tilde{A}$ and $\tilde{B}$ be two *LR* flat fuzzy numbers then

(i) $\tilde{A}\ge \tilde{B}$ if $R(\tilde{A})\ge R(\tilde{B})$
(ii) $\tilde{A}> \tilde{B}$ if $R(\tilde{A})>R(\tilde{B})$
(iii) $\tilde{A}=\tilde{B}$ if $R(\tilde{A})=R(\tilde{B})$

C. Linearity property of Yager's ranking index

Let $\tilde{A}=(m_1,n_1,\alpha_1,\beta_1)_{LR}$ and $\tilde{B}=(m_2,n_2,\alpha_2,\beta_2)_{LR}$ be two *LR* flat fuzzy numbers and k_1,k_2 be two non negative real numbers.

Using definition, the λ-cut $A_\lambda$and$B_\lambda$corresponding $\tilde{A}$ and $\tilde{B}$ are $A_\lambda = [m_1-\alpha_1 L^{-1}(\lambda), n_1+\beta_1 R^{-1}(\lambda)]$ and $B_\lambda=[m_2-\alpha_2 L^{-1}(\lambda), n_2+\beta_2 R^{-1}(\lambda)]$ using the property, $(\delta_1A_1 + \delta_2A_2)_\lambda= \delta1(A1)_\lambda + \delta2(A2)_\lambda$ ∀ δ_1, δ_2 ∈ (is the set of real numbers), the λ-cut $(k_1A +k_2A)$corresponding to $k_1\tilde{A} +k_2\tilde{B}$is

$(k_1A+k_2B)_\lambda= [k_1m_1 + k_2m_2 -(k_1\alpha_1+k_2\alpha_2)L^{-1}(\lambda),\ k_1n_1 +k_2n_2 +(k_1\beta_1 +k_2\beta_2)R^{-1}(\lambda)]$

usingYager's ranking index $R(k_1\tilde{A}+k_2\tilde{B})$ corresponding to fuzzy number $(k_1\tilde{A}+k_2\tilde{B})$ is:

International Journal of Computing Algorithm
Volume: 04 Issue: 02 December 2015 Pages: 76-78
ISSN: 2278-2397

$R(k_1\tilde{A}+k_2\tilde{B}) = \frac{1}{2}k_1(\int_0^1 m_1-\alpha_1 L^{-1}(\lambda)d\lambda) + \int_0^1 (n_1+\beta_1 R^{-1}(\lambda))d\lambda)$
$+ \frac{1}{2}k_2(\int_0^1 m_2-\alpha_2 L^{-1}(\lambda)d\lambda) + \int_0^1 (n_2+\beta_2 R^{-1}(\lambda))d\lambda)$
$=k_1R(\tilde{A})+k_2R(\tilde{B})$

Similarly it can be proved that $R(k_1\tilde{A}+k_2\tilde{B}) = k_1R(\tilde{A})+k_2R(\tilde{B})$

Table 1.Fuzzy assignment costs

Person→ Job↓	P_1	P_2	…	P_j	…	P_n
J_1	c_{11}	c_{12}	…	c_{ij}	…	c_{1n}
⋮	⋮	⋮	⋮	⋮	⋮	⋮
J_i	c_{j1}	c_{j2}	…	c_{jj}	…	c_{jn}
⋮	⋮	⋮	⋮	⋮	⋮	⋮
J_n	c_{n1}	c_{n2}	…	c_{nj}	…	c_{nn}

Where, $c_{ij} = (m_{ij},n_{ij},\alpha_{ij},\beta_{ij})_{LR}$

Example-Let us consider a fuzzy assignment problem rows representing 5 persons P_1, P_2, P_3, P_4, P_5 and columns representing 5 jobs Job1, job2, job3, job4, job5. The problem can find the minimal cost.

Table:2

C_{ij}	P_1	P_2	P_3	P_4	P_5
J_1	$(4,6,7,9)_{LR}$	$(3,4,6,9)_{LR}$	$(4,5,7,10)_{LR}$	$(3,5,6,7)_{LR}$	$(5,7,10,12)_{LR}$
J_2	$(4,7,10,14)_{LR}$	$(3,7,10,13)_{LR}$	$(2,3,10,12)_{LR}$	$(3,7,9,13)_{LR}$	$(4,10,13,15)_{LR}$
J_3	$(5,7,10,14)_{LR}$	$(2,4,10,13)_{LR}$	$(7,9,13,15)_{LR}$	$(5,7,12,15)_{LR}$	$(4,5,7,11)_{LR}$
J_4	$(7,9,13,14)_{LR}$	$(6,7,9,10)_{LR}$	$(7,9,10,13)_{LR}$	$(6,7,10,13)_{LR}$	$(7,9,10,12)_{LR}$
J_5	$(2,3,5,9)_{LR}$	$(5,6,7,10)_{LR}$	$(4,6,9,12)_{LR}$	$(5,7,9,13)_{LR}$	$(2,3,5,7)_{LR}$

Solution- First we show that how to calculate Y (4,6,7,9) by applying definition(3.2)Yager's Ranking method.

The membership value of first we choose L-R fuzzy number (4,6,7,9) is

$R(\tilde{A})={}^1/_2(\int_0^1 m-\alpha L^{-1}(\lambda)d\lambda) + \int_0^1 (n+\beta R^{-1}(\lambda))d\lambda)$

$R(c_{11})= {}^1/_2\,[(\int_0^1 4-7({}^1/_2)d\lambda) + \int_0^1 (6+9({}^1/_2)d\lambda)]$

$={}^1/_2\,[(\int_0^1 4-({}^7/_2)d\lambda) + \int_0^1 (6+({}^9/_2)d\lambda)]$

$={}^1/_2\,[(\int_0^1 ({}^{8}-{}^7/_2)d\lambda) + \int_0^1 ({}^{12}+{}^9/_2)d\lambda)]$

$={}^1/_2\,[(\int_0^1 ({}^1/_2)d\lambda) + \int_0^1 (({}^{21}/_2)d\lambda)]$

$={}^1/_2\,[(\int_0^1 ({}^{22}/_2)d\lambda)$

$={}^1/_2\,[(\int_0^1 (11\lambda)d\lambda)$

$=5.5$

Proceeding by similarly Yager'sRanking method

$R(c_{12})=2$, $R(c_{13})=2.67$, $R(c_{14})=6$, $R(c_{15})=6.5$,
$R(c_{21})=5.58$, $R(c_{22})=5.92$, $R(c_{23})=0.5$, $R(c_{24})=5.5$, $R(c_{25})=9.67$,
$R(c_{31})=5.25$, $R(c_{32})=2.67$,$R(c_{33})=8.5$,$R(c_{34})=5.5$,$R(c_{35})=2.38$
$R(c_{41})=11.75$, $R(c_{42})=7$,$R(c_{43})=5.67$,$R(c_{44})=3.67$,$R(c_{45})=8.5$,
$R(c_{51})=1.12$,$R(c_{52})=3.67$, $R(c_{53})=4.75$,$R(c_{54})=5.38$,
$R(c_{55})=1.75$.
$R(c^*_{12}) = R(c^*_{23}) = R(c^*_{35}) = R(c^*_{44}) = R(c^*_{51}) = 1$
$R(c^*_{11}) = R(c^*_{13}) = R(c^*_{14}) = R(c^*_{15}) = R(c^*_{21}) = R(c^*_{22}) = R(c^*_{23})$
$= R(c^*_{25}) = R(c^*_{31}) = R(c^*_{32}) = R(c^*_{33}) = R(c^*_{34}) = R(c^*_{41}) = R(c^*_{42}) =$
$R(c^*_{43}) = R(c^*_{45}) = R(c^*_{52}) = R(c^*_{53}) = R(c^*_{54}) = R(c^*_{55}) = 0$

Table:3 optimal table

C_{ij}	P_1	P_2	P_3	P_4	P_5
J_1	5.5	2	2.67	6	6.5
J_2	5.58	5.92	0.5	5.5	9.67
J_3	5.25	2.67	8.5	5.5	2.38
J_4	11.75	8	7	5.67	8.5
J_5	1.12	3.67	4.75	5.38	1.75

With optimal objective value $R(C_{ij}) = 11.67$
Which represents a optimal cost in other words the optimal assignment is

$J_1 \to P_2$, $J_2 \to P_3$, $J_3 \to P_5$, $J_4 \to P_4$, $J_5 \to P_1$
The fuzzy optimal cost is
$C_{12} + C_{23} + C_{35} + C_{44} + C_{15} = (3,4,6,9)_{LR} + (2,3,10,12)_{LR} + (4,5,7,11)_{LR} + (6,7,10,13)_{LR} + (2,3,5,9)_{LR}$
$= 2 + 0.5 + 2.38 + 5.67 + 1.12$
$= 11.67$

IV. CONCLUSION

Thus we have recalled some basic definitions and we have considered fuzzy assignment problem and introduced the yager's ranking method solution for these problems using triangular fuzzy numbers, trapezoidal fuzzy numbers, L-R fuzzy parameters and also, we have proposed a new algorithm for solving these problems directly by use of yager's ranking function.Finally, we have presented a method deal with sensitivity analysis of fuzzy assignment problem, and the presented method is illustrated with help of some numerical examples. In sensitivity analysis, the fuzzy optimal solution is again minimal cost, even though some variations are considered in the assignment problems. As this method consumes less time and is very easy to understand and apply, so it will be very helpful for decision makers who are dealing with logistic and supply chain problems.

International Journal of Computing Algorithm
Volume: 04 Issue: 02 December 2015 Pages: 76-78
ISSN: 2278-2397

REFERENCES

[1] L. A ,Zadeh, fuzzy sets, information and control, vol. 8,1965,pp. 338-353.
[2] R.Nagarajan and A.Solairaju computing improved fuzzy optimal Hungarian assignment problems with fuzzy cost under Robust Ranking Techniques IJCA vol.6-no.4 sep2010.
[3] A.Nagoorgani and V.N. Mohamed ,Solution of a Fuzzy Assignment Problem by using a New Ranking Method IJFMA vol.2, 2013, 8-16 , 6 june 2013.
[4] Andreae T. 2001. On the travelling salesman restricted to inputs satisfying a relaxedInequality. Networks, 38:59-67