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A Comparison of Various Normalization in Techniques for Order Performance by Similarity to Ideal Solution (TOPSIS)

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Abstract - The core of Operation Research is the development of approaches for optional Decision Making. A prominent class of such problem is Multi-Criteria Decision Making (MCDM). MCDM is used when different alternatives and different criteria are applied to make better Decision Making. There are several methodologies available in MCDM out which TOPSIS is one of the traditional methods in use. The TOPSIS Method is used to identify solution from a finite set of alternatives based upon simultaneous minimization of distance from a nadir point. In the first step of TOPSIS the vector normalization is performed. In our proposed work different normalization techniques are applied to find the best normalization which suits the TOPSIS Method. It is evaluated based on the performance measures like time and space complexity. To evaluate the proposed techniques the car selection is taken as a test case and the influence of normalization techniques in TOPSIS has been evaluated. Among the considered normalization techniques the performance of Linear Sum Based normalization technique achieves less computation time and space complexity.

Keywords — Multi Criteria Decision Making (MCDM), TOPSIS, Car Selection Problem, Normalization, Decision making.

I. INTRODUCTION

Since it was introduced in mid-1960s, multi-criteria decision making (MCDM) has become an important part of decision sciences[1], [2], [3]. It is mainly used to prescribe ways of evaluating, ranking and selecting the most favourable alternative(s) from a set of available ones which are characterized by multiple and usually conflicting criteria. However, the criteria are not always independent in some actual MCDM problems, and a possible relationship between a pair of criteria is the prioritization [4], [5], [6], [7], [8]. A typical example concerns the criteria of safety and cost in the cases of buying a car [9], selecting a bicycle for child [8] or air travel [7] etc.

In the above cases, we usually do not allow a loss in safety to be compensated by a benefit in cost, i.e., tradeoffs between safety and costs are unacceptable. Simply speaking, the criterion safety has a higher priority than cost. Moreover, according to Yager's (2008) [8] results, there may exist priority relationships among criteria in the problems of information retrieval [9].There are many MCDM techniques in use. The most commonly used MCDM techniques are: Technique for the Order of Prioritization by Similarity to Ideal Solution (TOPSIS), Analytic Hierarchy Process (AHP), Elimination EtChoixTraduisant la REalite´ (ELECTRE), PROMETHEE (Outranking) and VIKOR method.

One of the most widely used methods is TOPSIS (technique for order performance by similarity to ideal solution). It is a useful technique in dealing with multi-criteria decision making (MCDM) problems in the real world. TOPSIS is based on notation that the chosen alternative should have the shortest Euclidean distance from the ideal solution. It involves many numbers of steps among those steps normalization is an important step. The general calculation of the TOPSIS commence with the normalization. Because normalization is used to eliminate the units of each criterion so, that all the Criteria are dimensionless. Although, many normalization techniques are available TOPSIS uses vector normalization technique.

This research work has been proposed, to find the behavior of TOPSIS under different normalization techniques. To evaluate the proposed techniques the car selection problem is taken as a test case and the results are evaluated. The rest of the paper is set out as follows. In section 2, the relative literature are reviewed, section 3 describes about TOPSIS, section 4 applies different normalization technique in TOPSIS, section 5 describes about experimental design, section 6 presents the result and discussion, and finally ended up with the conclusion, the findings of the study and the future research .

II. PRIOR RESEARCH

According to Hwang .C.L & Yoon .K[10], TOPSIS uses vector normalization. Later it was proposed to employ linear normalization in the same multi- criteria method by Lai and Hwang, in 1994. Taking into consideration the notation that normalization procedure may affect the final MCDM solution, the TOPSIS algorithm that applies the criteria values transformation through normalization of vector, linear min-max, linear max, linear sum based and Gaussian are compared in this paper.

A. Overview of MCDM

Multi-Criteria Decision Making (MCDM) has been one of the fastest growing problem areas in many disciplines. The central problem is how to evaluate a set of alternatives in terms of a number of criteria. MCDM is nothing but selection of best actions from a set of alternative. The important task of MCDM

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is to find a “Good” Compromise. Zeleny in 1982 opens a book “Multiple Criteria Decision Making” with a statement: “It has become more and more difficult to see the world around us in a unidimensional way and to use only a single criterion when judging what we see”.

MCDM is used to evaluate a variety of courses of action against a set of established criteria. It can be defined as the alternative evaluation for selection and ranking purpose using qualitative and/or quantitative criteria which differ in measurements.
MCDM also referred as:

1. Multi-Criteria Decision Analysis (MCDA).
2. Multi-Dimensions Decision Making (MDDM).
3. Multi-Objective Decision Making (MODM).
4. Multi-Attributes Decision Making (MADM).

Alternatives: Alternatives represent the different choices of action or entities available to the decision maker. Usually, the set of alternatives is assumed to be finite, ranging from several to hundreds.
Criteria: Criteria represent the different dimensions from which alternatives can be viewed [6].

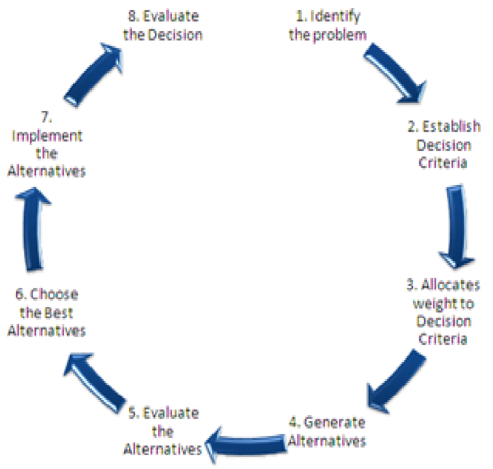

Figure 1: Decision Making Process

III. A DETAILED DESCRIPTION OF TOPSIS

TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) was first developed by Hwang &Yoon in 1981. The TOPSIS method assumes that each criterion has a tendency of monotonically increasing or decreasing utility. Therefore, it is easy to define the ideal and negative-ideal solutions. The Euclidean distance approach was proposed to evaluate the relative closeness of the alternatives to the ideal solution. Thus, the preference order of the alternatives can be derived by a series of comparisons of these relative distances.
The concept of TOPSIS is that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS).TOPSIS is the best method for ranking purpose [11]

Stepwise procedure for TOPSIS:
Step 1: Construct normalized decision matrix by using the vector normalization method.

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^{2}}} \quad \text{for} \quad i = 1, \ldots, m;$$
$$j = 1, \ldots, n \quad (1)$$

Where, a_{ij} is the original rating of the decision matrix and r_{ij} is the normalized value of the decision matrix.
Step 2: Construct the weighted normalized decision matrix by assigning different value (weight) to each criteria.

$$v_{ij} = w_j * r_{ij} \quad \text{for} \quad i = 1, \ldots, m;$$
$$j = 1, \ldots, n \quad (2)$$

Where w_j the weight for j is is the criterion.
Step 3: Determine the Positive Ideal Solution (PIS) & Negative Ideal Solution (NIS).

$$S^{*} = \{ v_{1j}^{*}, v_{2j}^{*}, v_{3j}^{*}, \ldots, v_{mj}^{*} \}, \quad (3)$$

Where $v_{ij}^{*} = \{ \max(v_{ij}) \text{ if } j \in J; \min(v_{ij}) \text{ if } j \in J^{-} \}$

$$S^{-} = \{ v_{1j}^{-}, v_{2j}^{-}, v_{3j}^{-}, \ldots, v_{mj}^{-} \}, \quad (4)$$

Where $v_{ij}^{-} = \{ \min(v_{ij}) \text{ if } j \in \ ; \max(v_{ij}) \text{ if } j \in J^{-} \}$

Step 4: Calculate the Distance of each alternative by applying the Euclidean Distance method.
The distance from Positive Ideal Solution is:

$$D_i^{*} = \sqrt{\sum_{j=1}^{n} \left(v_{ij}^{*} - S^{*}\right)^{2}}$$
$$\text{for} \quad i = 1, \ldots, m \qquad (5)$$

Similarly, the distance from Negative Ideal Solution is:

$$D_i^{-} = \sqrt{\sum_{j=1}^{n} (v_{ij}^{-} - S^{-})^{2}}$$
$$\text{for } i = 1, \ldots, m \qquad (6)$$

Step 5: Calculate the relative closeness to the ideal solution C_i .

$$C_i = \frac{D_i^{*}}{D_i^{-} + D_i^{*}}$$
$$\text{for } i = 1, 2, \ldots, m \;\&\; 0 < C_i < 1 \; (7)$$

Step 6: Rank the alternatives by selecting the alternative with C_i closest to 1.

IV. COMPARISON OF VARIOUS NORMALIZATION TECHNIQUES

In TOPSIS method, the calculation of each alternative distance from the Positive Ideal Solution and the Negative Ideal Solution draws attention.

Decision Matrix:
A MCDM problem can be expressed in a matrix format. A decision matrix A is an (M*N) matrix in which the element a_{ij} indicates the performance of alternative A_i when it is evaluated in terms of decision criterion C_j, (for i=1,2,…,M & j=1,2,…,N).

International Journal of Computing Algorithm
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$$A = \begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ A_1 & a_{11} & a_{12} & \cdots & a_{1n} \\ A_2 & a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ A_m & a_{m1} & a_{m2} & \cdots & a_{mn} \end{array}$$

Alternatives	Criteria			
	Style	Reliability	Fuel-economy	Cost
Civic Coupe	7	9	9	8
Saturn Coupe	8	7	8	7
Ford Escort	9	6	8	9
Mazda Miata	6	7	8	6

Table 1: Original decision matrix

Normalization:
The process of normalizing the ratings of different alternatives into a same range is termed as 'Normalization'. Normalization is mainly used to eliminate the units of each criterion, so that all the criteria are dimensionless.
There are different methods for normalization and these methods are used to obtain concise answer. A comparative analysis is carried out on the five well known normalization techniques is briefly illustrated below.

- Vector Normalization
- Linear Max-Min Normalization
- Linear Sum based Normalization
- Linear Max Normalization
- Gaussian Normalization

A. *Vector Normalization*

The original decision matrix can be normalized using the vector normalization method.

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}} \quad \text{for} \quad i = 1, \ldots, m; j = 1..n$$

Where a_{ij} - Original rating of the decision matrix,
r_{ij} - Normalized value of the decision matrix.

Alternatives	Distance D^*	Distance D^-	Relative Closeness Coefficients	Rank
Civic Coupe	0.029	0.084	0.74	1
Saturn Coupe	0.058	0.040	0.41	3
Ford Escort	0.091	0.019	0.17	4
Mazda Miata	0.059	0.047	0.44	2

Table 2 : The distances of alternatives to PIS & NIS, the relative closeness coefficients and the ranking based on Vector Normalization.

B. *Linear Max-Min Normalization*

The Linear Max-Min Normalization technique has the following general form:
The normalized value r_{ij} for benefit criteria is obtained by

$$r_{ij} = \frac{a_{ij} - a_j^{min}}{a_j^{max} - a_j^{min}} \quad \text{for} \quad i = 1, \ldots, m$$

$j = 1, \ldots, n$

The normalized value r_{ij} for cost criteria is computed as

$$r_{ij} = \frac{a_j^{max} - a_{ij}}{a_j^{max} - a_j^{min}} \quad \text{for} \quad i = 1, \ldots, m;$$

$j = 1, \ldots, n$

Where a_j^{max} - Maximum rating of the alternatives for each criterion C_j *(j = 1, 2, …, n)*
a_j^{min} - Minimum rating of the alternatives for each criterion C_j *(j = 1, 2, …, n)*.

Alternatives	Distance D^*	Distance D^-	Relative Closeness Coefficients	Rank
Civic Coupe	0.067	0.50	0.88	1
Saturn Coupe	0.40	0.16	0.28	4
Ford Escort	0.50	0.22	0.31	3
Mazda Miata	0.44	0.36	0.45	2

Table 3 : The distances of alternatives to PIS & NIS, the relative closeness coefficients and the ranking based on Linea Min-Max Normalization.

C. *Linear Sum Based Normalization*

Normalize the decision matrix by using the Linear Sum based Normalization technique. This method divides the rating of each alternative by the sum of rating of each criterion as follows

$$r_{ij} = \frac{a_{ij}}{\sum_{j=1}^{n} a_j} \quad \text{for} \quad i = 1, \ldots, m;$$

$j = 1, \ldots, n$

Where a_j - Performance rating of each alternative for criteria C_j *(j = 1, 2, …, n)*.

Alternatives	Distance D^*	Distance D^-	Relative Closeness Coefficients	Rank
Civic Coupe	0.007	0.044	0.38	1
Saturn Coupe	0.042	0.015	0.26	3
Ford Escort	0.749	0.007	0.009	4
Mazda Miata	0.043	0.021	0.32	2

Table 4 : The distances of alternatives to PIS & NIS, the relative closeness coefficients and the ranking based on Linea Sum-Based Normalization.

D. *Linear Max Normalization*

The Linear Max Normalization technique has the following general form:

International Journal of Computing Algorithm
Volume: 03 Issue: 03 December 2014 Pages: 255-259
ISSN: 2278-2397

The normalized value r_{ij} for benefit criteria is determined by

$$r_{ij} = \frac{a_{ij}}{a_j^{max}} \quad \text{for} \quad i = 1, \dots, m; \quad j = 1, \dots, n$$

The normalized value r_{ij} for cost criteria is computed as

$$r_{ij} = 1 - \frac{a_{ij}}{a_j^{max}} \quad \text{for} \quad i = 1, \dots, m; \quad j = 1, \dots, n$$

Where a_j^{max}-Maximum rating of the alternatives for each criterion $C_j (j = 1,2,\dots, n)$.

Alternatives	Distance D^*	Distance D^-	Relative Closeness Coefficients	Rank
Civic Coupe	0.031	0.138	0.26	4
Saturn Coupe	0.105	0.049	0.31	1
Ford Escort	0.136	0.056	0.29	3
Mazda Miata	0.105	0.045	0.3	2

Table 5 : The distances of alternatives to PIS & NIS, the relative closeness coefficients and the ranking based on Linea Max Normalization.

E. Gaussian Normalization

Normalize the decision matrix by using the Gaussian Normalization technique. Two factors are taken into consideration that leads to the rating variance of the users with similar interests.

a) Average rating shift

b) Different rating scales

The above two ideas are combined together, to normalize the rating of each alternative
i based on the criteria j is computed by:

$$r_{ij} = \frac{a_{ij} - \bar{a}_i}{\sqrt{\sum_{j=1}^{n} \left(a_{ij} - \bar{a}_i\right)^2}}$$

for $i = 1, \dots, m; \quad j = 1, \dots, n$

Where a_{ij} - represent the original rating of each alternative based on the criteria. $\bar{a}_i$- stands for the average rating of alternative i.

Alternatives	Distance D^*	Distance D^-	Relative Closeness Coefficients	Rank
Civic Coupe	0.044	0.132	0.75	1
Saturn Coupe	0.090	0.025	0.45	3
Ford Escort	0.088	0.052	0.02	4
Mazda Miata	0.061	0.033	0.62	2

Table 6 : The distances of alternatives to PIS & NIS, the relative closeness coefficients and the ranking based on Gaussian Normalization.

V. EXPERIMENTAL DESIGN

The efficiency of various normalization techniques can be determined by the following two metrics.

a) Computation time of the algorithm (Time Complexity)

b) Space required by the algorithm (Space Complexity)

METHODS	TIME COMPLEXITY (Seconds)	SPACE COMPLEXITY (Bytes)
Vector Normalization	0.003984	3600
Linear Max-Min Normalization	0.005687	3856
Linear Max Normalization	0.003026	2999
Linear Sum based Normalization	0.001464	2336
Gaussian Normalization	0.001996	2988

Table 7: Complexities of various Normalization Techniques

VI. RESULT AND DISCUSSION

Table 7 describes the Time and Space Complexity of various normalization techniques, from the above table it is observed that the Linear Sum based Normalization technique achieves less computation time and Space Complexity. The above techniques are applied in the banking sector and the performance of the various normalization techniques are measured using time and space complexity.

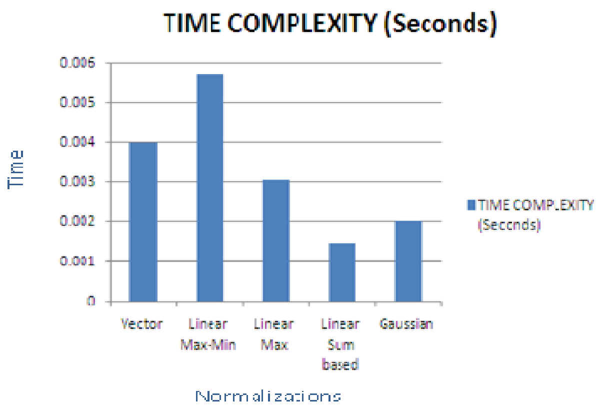

Figure 2: Comparison of various normalizations based on Time Complexity

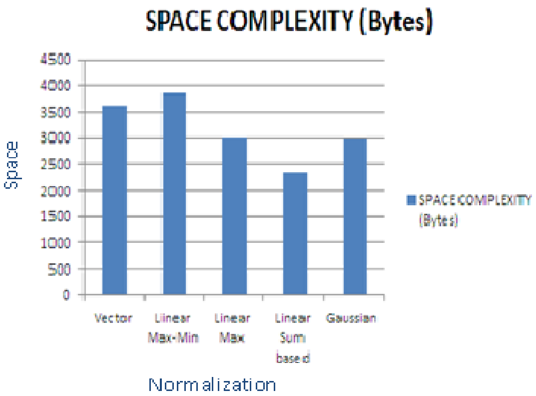

Figure 3: Comparison of various normalizations based on Space Complexity

Figure 2 shows diagrammatical view of linear sum based normalization which has less time complexity. Where, figure 3

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shows diagrammatical view of linear sum based normalization which has less space complexity than compared normalizations

VII. CONCLUSION

This study proposes the best normalization technique to be used in the TOPSIS for the optimal solution in MCDM. TOPSIS is a practical and useful technique for ranking and selection of a number of externally determined alternatives through distance measures. First step of the TOPSIS is normalization. In this paper, the top five normalization techniques namely vector, linear max-min, linear max, linear sum based and Gaussian normalizations are applied in TOPSIS and further TOPSIS steps are proceeded. The car selection problem is taken as a test case and the influence of normalization technique in TOPSIS has been evaluated and tabulated. Time and space complexity are calculated for each and every compared technique with the help of Matlab. The comparison among the result concludes that, linear sum based normalization acquired less time and space than others. If it used in the TOPSIS for decision making, the optimal solution can be obtained. Future research may try to extend, to apply the different normalization technique in other MCDM problems.

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