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Few More Results on Sum Labeling of Split Graphs

J.Gerard Rozario
Department of Mathematics, IFET College of Engineering, Villupuram, Tamil Nadu, India
Email: rozario.gerard@yahoo.com

Abstract- A sum labeling is a mapping λ from the vertices of G into the positive integers such that, for any two vertices u, v ϵ V (G) with labels λ(u) and λ(v), respectively, (uv) is an edge iff λ(u) + λ(v) is the label of another vertex in V (G). Any graph supporting such a labeling is called a sum graph. It is necessary to add (as a disjoint union) a component to sum label a graph. This disconnected component is a set of isolated vertices known as isolates and the labeling scheme that requires the fewest isolates is termed optimal. The number of isolates required for a graph to support a sum labeling is known as the sum number of the graph. In this paper, we will obtain optimal sum labeling scheme for path union of split graph of star, $K_{1,m} \odot Spl(P_n)$ and $K_{1,m} \odot Spl(K_{1,n})$.

Keywords: Sum Labeling, Sum graph, Sum number, split graph, path union. 2010 AMS Subject Classification: 05C78

I. INTRODUCTION

All the graphs considered here are simple, finite and undirected. For all terminologies and notations we follow Harary [1] and graph labeling as in [2]. Sum labeling of graphs was introduced by Harary [3] in 1990. Following definitions are useful for the present study.

Definition 1.1 *Sum Labeling* is a mapping λ from the vertices of G into the positive integers such that, for any two vertices u, v ϵ V (G) with labels λ(u) and λ(v), respectively, (uv) is an edge iff λ(u) + λ(v) is the label of another vertex in V (G). Any graph supporting such a labeling is called a *Sum Graph*.

Definition 1.2 It is necessary to add (as a disjoint union) a component to sum label a graph. This disconnected component is a set of isolated vertices known as *Isolates* and the labeling scheme that requires the fewest isolates is termed *Optimal*.

Definition 1.3 The number of isolates required for a graph G to support a sum labeling is known as the *Sum Number* of the graph. It is denoted as $\sigma(G)$.

Definition: 1.4 (Shiama [4]) For a graph G the split graph is obtained by adding to each vertex v, a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G. The resultant graph is called the Split Graph denoted by Spl (G).

Definition 1.5 (Shee and Ho. et al [5]) Let $G_1, G_2, \ldots, G_n$, $n \geq 2$ be *n* copies of a fixed graph G. The graph obtained by adding an edge between G_i and G_{i+1} for i =1, 2,..., n-1 is called *path union* of *G*.

II. SUM LABELING FOR SPLIT GRAPHS

In [6], Gerard et. al has proved that split graph of path, star are sum graph with sum number 1 and bi – star is sum graph with sum number 2.

Theorem: 2.1 Path union of split graph of star $K_{1,n}$ is a sum graph with sum number 1.

Proof: Consider a star $K_{1,n}$ with (n+1) vertices. Let G be the split graph of star, Spl ($K_{1,n}$). Let G* be the path union of m copies of G. Let $v_1, v_{11}, v_{12}, \ldots, v_{1n}, v_2, v_{21}, v_{22}, \ldots, v_{2n}, \ldots, v_m, v_{m1}, v_{m2}, \ldots, v_{mn}$ be the vertices of m copies of the star $K_{1,n}$. Let $u_1, u_{11}, u_{12}, \ldots u_{1n}, u_2, u_{21}, u_{22}, \ldots, u_{2n}, \ldots, u_m, u_{m1}, u_{m2}, \ldots, u_{mn}$ be the vertices corresponding to $v_1, v_{11}, v_{12}, \ldots, v_{1n}, v_2, v_{21}, v_{22}, \ldots, v_{2n}, \ldots, v_m, v_{m1}, v_{m2}, \ldots, v_{mn}$ of m copies of the star $K_{1,n}$ which are added, to obtain the split graph of m copies of star. G* has $3nm$ vertices and $3nm + (m-1)$ edges. Let x be the isolated vertex.

Define f: V (G*) → N

$$f(v_1) = 1$$
$$f(v_2) = 2$$
$$f(v_i) = f(v_{(i-1)})f(v_{(i-2)}) \; for \; 3 \leq i \leq m$$
$$f(v_{11}) = f(v_m) + f(v_{(m-1)})$$
$$for \; 1 \leq i \leq m$$

$$\begin{cases} f(v_{ij}) = f(v_{i(j-1)}) + f(v_i) & for \;\; 2 \leq j \leq n \\ f(u_i) = f(v_{in}) + f(v_i) & \\ f(u_{i1}) = f(u_i) + f(v_{i1}) & \\ f(u_{ij}) = f(u_{i(j-1)}) + f(v_i) & for \;\; 2 \leq j \leq n \\ f(v_{(i+1)1}) = f(u_{in}) + f(v_i) & if \;\; i \neq m \end{cases}$$
$$f(x) = f(u_{mn}) + f(v_m)$$

Thus, Path union of Split graph of star $K_{1,n}$ is a sum graph with sum number 1.

Illustration: Sum labeling for path union of split graph of star $K_{1,n}$ is given in figure 2.1

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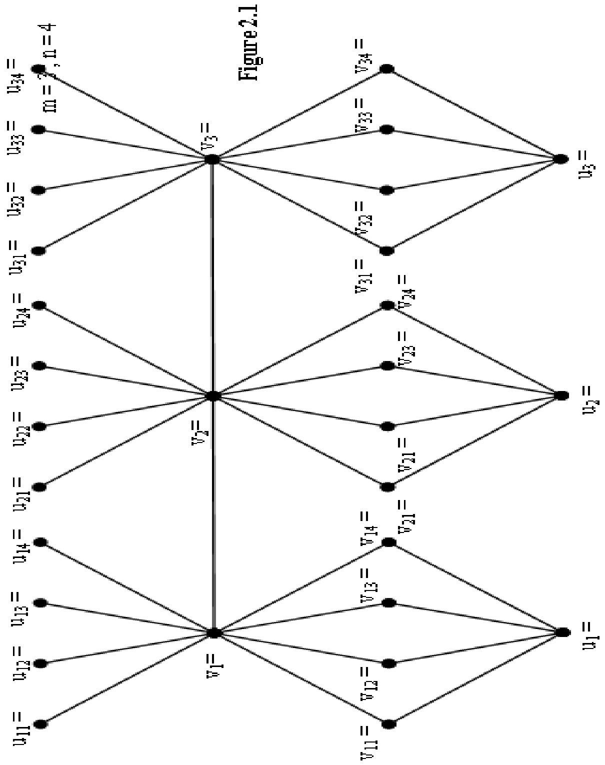

Figure 2.1

$K_{1,m} \odot Spl(P_n)$

$K_{1,m} \odot Spl(P_n)$ is obtained by attaching a copy of $Spl(P_n)$ to each pendent vertex of $K_{1,m}$.

Theorem: 2.2 $K_{1,m} \odot Spl(P_n)$ is a sum graph with sum number 1.

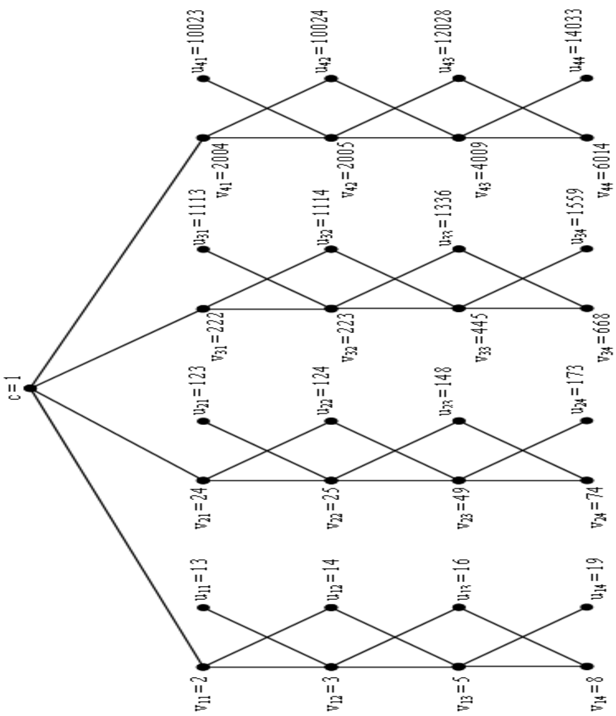

Figure 2.2

$$f(x) = f(u_{mn}) + f(v_{m(n-1)})$$

Hence, $K_{1,m} \odot Spl(P_n)$ is a sum graph with sum number 1.

Illustration: Sum labeling for $K_{1,m} \odot Spl(P_n)$ *is given in figure 2.2*

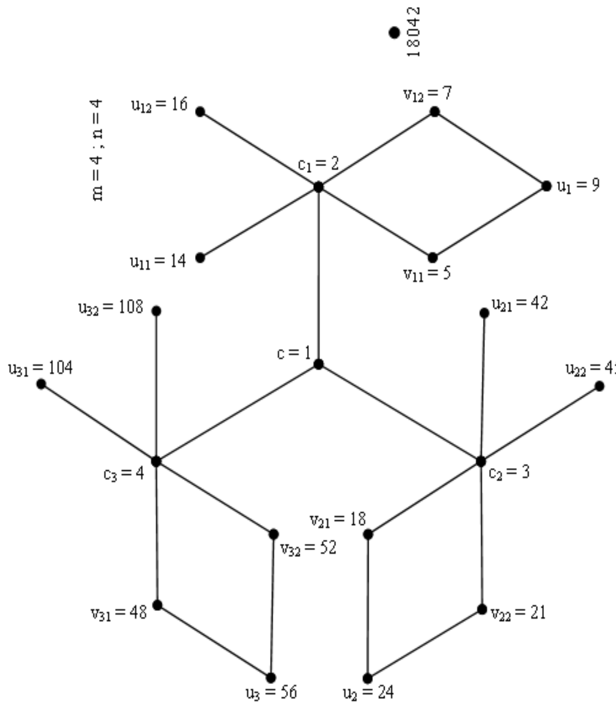

Figure 2.3

Proof: Let c, v_1, v_2,……, v_m be the vertices of $K_{1,m}$ where c is the centre of the star. Let v_{11}, v_{12}, ……, v_{1n}, u_{11}, u_{12}, ……u_{1n}, v_{21}, v_{22}, ……,v_{2n}, u_{21}, u_{22}, ……,u_{2n},……, v_{m1}, v_{m2}, ……, v_{mn}, u_{m1}, u_{m2}, ……, u_{mn} be the vertices of the m copies of the split graph of path P_n. The vertices v_{11}, v_{21},……, v_{m1} are attached to the vertices v_1, v_2,……, v_m respectively. Let G = $K_{1,m} \odot Spl(P_n)$. Therefore the vertex set of G, V(G) = {c, v_{11}, v_{12}, ……, v_{1n}, u_{11}, u_{12}, ……u_{1n}, v_{21}, v_{22}, ……,v_{2n}, u_{21}, u_{22}, ……,u_{2n},……, v_{m1}, v_{m2}, ……, v_{mn}, u_{m1}, u_{m2}, ……, u_{mn}}. G has $2nm + 1$ vertices and $3n(n-1) + m$ edges. Let x be the isolated vertex.

Define f: V (G) → N

$$f(c) = 1 \qquad f(v_{11}) = 2$$

$$for\ 1 \le i \le m$$

$$\begin{cases} f(v_{i2}) = f(v_{i1}) + 1 \\ f(v_{ij}) = f(v_{i(j-1)}) + f(v_{i(j-2)}) \ \ for\ 3 \le j \le n \\ f(u_{i1}) = f(v_{i(n-1)}) + f(v_{in}) \\ f(u_{i2}) = f(u_{i1}) + 1 \\ f(u_{ij}) = f(u_{i(j-1)}) + f(v_{i(j-2)}) \ \ for\ 3 \le j \le n \\ f(v_{(i+1)1}) = f(u_{in}) + f(v_{i(n-1)}) \qquad if\ i \ne m \end{cases}$$

$K_{1,m} \odot Spl(K_{1,n})$

$K_{1,m} \odot Spl(K_{1,n})$ is obtained by attaching a copy of $Spl(K_{1,n})$ to each pendent vertex of $K_{1,m}$.

Theorem: 2.3 $K_{1,m} \odot Spl(K_{1,n})$ is a sum graph with sum number 1.

Proof: Let c, $v_1, v_2, \ldots, v_m$ be the vertices of $K_{1,m}$ where c is the centre of the star. Let $c_1, v_{11}, v_{12}, \ldots, v_{1n}, u_1, u_{11}, u_{12}, \ldots u_{1n}, c_2, v_{21}, v_{22}, \ldots, v_{2n}, u_2, u_{21}, u_{22}, \ldots, u_{2n}, \ldots, c_m, v_{m1}, v_{m2}, \ldots, v_{mn}, u_m, u_{m1}, u_{m2}, \ldots, u_{mn}$ be the vertices of the m copies of the split graph of star $K_{1,n}$. The vertices $c_1, c_2, \ldots, c_m$ are attached to the vertices $v_1, v_2, \ldots, v_m$ of $K_{1,n}$ respectively. Let G = $K_{1,m} \odot Spl(K_{1,n})$. Therefore the vertex set of G, V(G) = $\{c, c_1, v_{11}, v_{12}, \ldots, v_{1n}, u_1, u_{11}, u_{12}, \ldots u_{1n}, c_2, v_{21}, v_{22}, \ldots, v_{2n}, u_2, u_{21}, u_{22}, \ldots, u_{2n}, \ldots, c_m, v_{m1}, v_{m2}, \ldots, v_{mn}, u_m, u_{m1}, u_{m2}, \ldots, u_{mn}\}$. G has $2nm + 1$ vertices and $3n(n-1) + m$ edges. Let x be the isolated vertex.

Define f: V (G) → N

$$f(c) = 1 \qquad f(c_1) = 2$$
$$f(c_i) = f(c_{(i-1)}) + 1 \quad for \ 2 \le i \le m$$
$$f(v_{11}) = f(c_m) + 1$$
$$for \ 1 \le i \le m$$

$$\begin{cases} f(v_{ij}) = f(v_{i(j-1)}) + f(v_i) \ for \ 2 \le j \le n \\ f(u_i) = f(v_{in}) + f(c_i) \\ f(u_{i1}) = f(u_i) + f(v_{i1}) \\ f(u_{ij}) = f(u_{i(j-1)}) + f(c_i) \quad for \ 2 \le j \le n \\ f(v_{(i+1)1}) = f(u_{in}) + f(c_i) \quad if \ i \ne m \end{cases}$$

$$f(x) = f(u_{mn}) + f(c_m)$$

Thus, $K_{1,m} \odot Spl(K_{1,n})$ is a sum graph with sum number 1.

Illustration: Sum labeling for $K_{1,m} \odot Spl(K_{1,n})$ *is given in figure 2.3*

m = 3 ; n = 2

REFERENCES

[1] Harary F, Graph theory, Addison Wesley, Reading, Massachusetts, 1972.
[2] Gallian J A, A dynamic survey of graph labeling, The Electronics Journal of Combinatorics, 16, (2009) DS6.
[3] Harary F, Sum graphs and Difference graphs, Congress Numerantium, no.72, 101-108, 1990
[4] Shiama. J, Permutation Labeling for some shadow graphs, International Journal of Computer Applications (0975 – 8887), Volume 40 – No.6, February 2012.
[5] S.C. Shee , Y. S Ho (1996), "The cardiality of Path-union of n copies of a Graph ", Discrete Math, 151, 221-229
[6] Gerard Rozario J, Jon Arockiaraj J and Lawrence Rozario Raj. P, Sum Labeling for some Split graphs, International Journal of Computational and Applied Maths, in press.