

An Application of Interval Valued Fuzzy Matrices for Finding the Disease in Plants

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Abstract- In this paper, we extend Sanchez's approach in agricultural field to find the disease of the plant by using the representation of an interval valued fuzzy matrix as an interval matrix of two fuzzy matrices.

Keywords- Fuzzy matrix, Interval Valued Fuzzy Matrix (IVFM)

I. INTRODUCTION

The concept of Interval Valued fuzzy matrix (IVFM) is one of the recent topics developed for dealing with the uncertainties present in most of our real life situations. The parameterization tool of Interval valued fuzzy matrix enhances the flexibility of its applications. Most of our real life problems in Agriculture fertilizer, management environment and Social Sciences often involve data which are not necessarily crisp, precise and deterministic in character due to various uncertainties associated with these problems, such uncertainties are usually being handled with the help of the topics like probability, fuzzy sets, intuitionistic fuzzy sets, interval mathematics and rough sets etc. The concept of IVFM as a generalization of fuzzy matrix was introduced and developed by Shyamal and pal [4], by extending the max. min operations on Fuzzy algebra $F = [0,1]$, for elements $a, b \in F$, $a+b = \max\{a,b\}$ and $a \cdot b = \min\{a,b\}$. Let F_{mn} be the set of all $m \times n$ Fuzzy Matrices over the Fuzzy algebra with support $[0,1]$. We have represented an IVFM $A = (a_{ij}) = ([a_{ijL}, a_{ijU}])$ where each a_{ij} is a subinterval of the interval $[0,1]$, as the Interval matrix $A = [A_L, A_U]$ whose ij^{th} entry is the interval $[a_{ijL}, a_{ijU}]$, where the lower limit $A_L = (a_{ijL})$ and the upper limit $A_U = (a_{ijU})$ are fuzzy matrices such that $A_L \leq A_U$.

II. PRELIMINARIES

A. Definition

Let a matrix $A = [a_{ij}]_{m \times n}$ where $a_{ij} \in [0,1]$, $1 \leq i \leq m$ and $1 \leq j \leq n$. Then A is fuzzy matrix. Note:- All fuzzy matrices are matrices but every matrix is not a fuzzy matrix.

B. Definition

The IVFM denote the set of all interval valued fuzzy matrices, that is, fuzzy matrices whose entries are all subintervals of the interval $[0,1]$.

C. Definition

For a pair of fuzzy matrices $[E, F]$ where E & F are fuzzy $m \times n$ matrices. i.e., $E = (e_{ij})$, $F = (f_{ij})$ and $e_{ij}, f_{ij} \in [0,1]$ further $e_{ij} \leq f_{ij}$ for $1 \leq i \leq m$ and $1 \leq j \leq n$. Thus the fuzzy interval matrices or interval fuzzy matrices contain all matrices $G = (g_{ij})$ such that (g_{ij}) takes its value in between e_{ij} & f_{ij} . i.e, $G \in [E, F]$ implies

$e_{ij} \leq g_{ij} \leq f_{ij}$. The interval matrix denoted as $[E, F]$, whose ij^{th} entry is the interval with lower limit e_{ij} and upper limit f_{ij} , that is $[e_{ij}, f_{ij}]$. The basic operation on IVFM

For $A = (a_{ij}) = ([a_{ijL}, a_{ijU}])$ and

$B = (b_{ij}) = ([b_{ijL}, b_{ijU}])$ of order $m \times n$ their sum denoted as $A + B$ is defined as

$$A + B = (a_{ij} + b_{ij}) = [(a_{ijL} + b_{ijL}), (a_{ijU} + b_{ijU})]$$

Then addition is,

$$A + B = \max\{a_{ij}, b_{ij}\} = [\max\{a_{ijL}, b_{ijL}\}, \max\{a_{ijU}, b_{ijU}\}] \quad (2.1)$$

For $A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{n \times p}$ their product denoted as AB is defined as

$$AB = (c_{ij}) = \sum_{k=1}^n a_{ik} \cdot b_{kj}, i=1, 2, \dots, m$$

$$j=1, 2, \dots, p$$

$$= \left[\sum_{k=1}^n (a_{ikL} \cdot b_{kjL}), \sum_{k=1}^n (a_{ikU} \cdot b_{kjU}) \right]$$

$$= [\max \min(a_{ikL} \cdot b_{kjL}), \max \min(a_{ikU} \cdot b_{kjU})]$$

(2.2)

If $A = [A_L, A_U]$ and $B = [B_L, B_U]$

Then,

$$A + B = [A_L + B_L, A_U + B_U]$$

$$AB = [A_L B_L, A_U B_U] \quad (2.3)$$

$A \geq B$ if and only if $a_{ijL} \geq b_{ijL}$ and $a_{ijU} \geq b_{ijU}$ if and only if $A+B=A$ (2.4)

In particular if $a_{ijL} = a_{ijU}$ and $b_{ijL} = b_{ijU}$ Then (2.1) reduces to the standard max.min composition of Fuzzy matrices.

III. APPLICATION OF INTERVAL VALUED FUZZY MATRICES FOR FINDING THE DISEASE IN PLANTS.

Suppose S is a set of plant growth, D is a set of diseases and C is a set of cultivation, construct an interval valued fuzzy matrix (F, D) over S , where F is a mapping $F: D \rightarrow \tilde{F}(s)$, $\tilde{F}(s)$ is a set of all interval valued fuzzy sets of S . A relation matrix, R_1 is constructed from the interval valued fuzzy matrix (F, D) and called plant growth – disease matrix. Similarly its compliment $(F, D)^c$ gives another relation matrix, say R_2 , called non plant growth – disease matrix. Again we construct another interval valued fuzzy matrix (F_1, S) over C , where F_1 is a mapping given by $F_1: S \rightarrow \tilde{F}(c)$. This interval valued fuzzy matrix gives another relation matrix Q called cultivation plant growth matrix. Then we obtain two new relation matrices $T_1 = Q R_1$

and $T_2 = Q \cdot R_2$ called plant growth cultivation matrix and non plant growth cultivation matrix respectively.

$$\text{Now, } T_1 = Q \cdot R_1 \quad (3.1)$$

$$T_2 = Q \cdot R_2 \quad (3.2)$$

Let $T_1 = [T_{1L}, T_{1U}]$, $Q = [Q_L, Q_U]$, $R_1 = [R_{1L}, R_{1U}]$, $T_2 = [T_{2L}, T_{2U}]$ and $R_2 = [R_{2L}, R_{2U}]$, be the representation of the form (2.1) for the IVFM T_1 , Q , R_1 , T_2 and R_2 . Then by using the IVFM operation (2.4) in (3.1) and (3.2) we get,

$$T_{1L} = Q_L \cdot R_{1L} \text{ and } T_{1U} = Q_U \cdot R_{1U} \quad (3.3)$$

$$T_{2L} = Q_L \cdot R_{2L} \text{ and } T_{2U} = Q_U \cdot R_{2U} \quad (3.4)$$

Let us define the Non – disease matrices T_{3L} , T_{3U} , T_{4L} and T_{4U} corresponding to T_{1L} , T_{1U} , T_{2L} and T_{2U} respectively as

$$T_{3L} = Q_L \cdot (J - R_{1L}) \text{ and } T_{3U} = Q_U \cdot (J - R_{1U}) \quad (3.5)$$

$$T_{4L} = Q_L \cdot (J - R_{2L}) \text{ and } T_{4U} = Q_U \cdot (J - R_{2U}) \quad (3.6)$$

Where J is the matrix with all entries '1' now

$$S_{T1L} = \max_{i,j} [T_{1L}(c_i, d_j), T_{4L}(c_i, d_j)] \text{ and } S_{T1U} = \max_{i,j} [T_{1U}(c_i, d_j), T_{4U}(c_i, d_j)] \quad (3.7)$$

$$\forall i=1,2,3 \text{ and } j=1,2$$

$$S_{T2L} = \max_{i,j} [T_{2L}(c_i, d_j), T_{3L}(c_i, d_j)] \text{ and}$$

$$S_{T2U} = \max_{i,j} [T_{2U}(c_i, d_j), T_{3U}(c_i, d_j)] \quad (3.8)$$

$$\forall i=1,2,3 \text{ and } j=1,2$$

We calculate the score S_{T1} and S_{T2} for and against the diseases respectively.

$$S_{T1} = \max_{i,j} [S_{T1U}(c_i, d_j), S_{T1L}(c_i, d_j)] \quad (3.9)$$

$$\forall i=1,2,3 \text{ and } j=1,2$$

$$S_{T2} = \max_{i,j} [S_{T2U}(c_i, d_j), S_{T2L}(c_i, d_j)] \quad (3.10)$$

$$\forall i=1,2,3 \text{ and } j=1,2$$

$$\text{Now if } \max_j [S_{T1}(c_i, d_j) - S_{T2}(c_i, d_j)] \quad (3.11)$$

occurs for exactly (c_i, d_k) only, then we conclude that the acceptable hypothesis for cultivation c_i is the disease d_k . In case there is a tie, the process has to be repeated for cultivation c_i by reassessing the plant growth.

A. Algorithm

- Input the interval valued fuzzymatrices (F, D) and $(F, D)^C$ over the set S of plant growth where D is the set of diseases. Also write the Agriculture knowledge matrix R_1 and R_2 representing the relation
- matrices of the IVFM (F, D) and $(F, D)^C$ respectively.
- $R_2 = 1 - R_1 = [1 - R_{1U}, 1 - R_{1L}]$
- Input the IVFM (F_1, S) over the set C of cultivation and write its relation matrix Q .
- To find the relation matrices
- $T_{1L} = Q_L \cdot R_{1L}$ and $T_{1U} = Q_U \cdot R_{1U}$
- (ii) $T_{2L} = Q_L \cdot R_{2L}$ and $T_{2U} = Q_U \cdot R_{2U}$
- Then we get
- (iii) $T_{3L} = Q_L \cdot (J - R_{1L})$ and

- $T_{3U} = Q_U \cdot (J - R_{1U})$
- (iv) $T_{4L} = Q_L \cdot (J - R_{2L})$ and
- $T_{4U} = Q_U \cdot (J - R_{2U})$
- To find S_{T1L} , S_{T1U} , S_{T2L} and S_{T2U}
- To find the Scores S_{T1} and S_{T2}

Find

$$S_K = \max_j \{S_{T1}(c_i, d_j) - S_{T2}(c_i, d_j)\}$$

Then we conclude that the cultivation c_i is affected from the disease d_k .

(8) If $S_k > 1$ then go to step one and repeat the process, if $S_k < 1$ then stop. Case Study - Suppose there are three cultivations paddy, wheat and moongdal in an Agriculture with plant growths Dwarf plant, Tall plant, no leaf, no growth problem. Let the possible diseases relating to the above plant growth be tritcale disease and nematodes. We consider the set $S = \{e_1, e_2, e_3, e_4\}$ as universal set, where e_1, e_2, e_3 , and e_4 represent the plant growth Dwarf plant, Tall plant, no leaf, no growth problem respectively and the set $D = \{d_1, d_2\}$ where d_1 and d_2 represent the parameters tritcale and nematodes respectively.

Suppose that,

$$F(d_1) = \{ \langle e_1, [0.7, 1] \rangle, \langle e_2, [0.1, 0.4] \rangle, \langle e_3, [0.5, 0.6] \rangle, \langle e_4, [0.2, 0.4] \rangle \}$$

$$F(d_2) = \{ \langle e_1, [0.6, 0.9] \rangle, \langle e_2, [0.4, 0.6] \rangle, \langle e_3, [0.3, 0.6] \rangle, \langle e_4, [0.8, 0.1] \rangle \}$$

The Interval valued fuzzy matrix (F, D) is a parameterized family $[F(d_1), F(d_2)]$ of all interval valued fuzzy matrix over the set S and are determined from expert Agriculture documentation. Thus the fuzzy matrix (F, D) gives an approximate description of the interval valued fuzzy matrix agriculture knowledge of the two diseases and their plant growth. This interval valued fuzzy matrix (F, D) and its complement $(F, D)^C$ are represented by two relation matrices R_1 and R_2 called plant growth – disease matrix and non plant growth disease matrix respectively. given by,

$$R_1 = \begin{matrix} & d_1 & d_2 \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} & \begin{bmatrix} [0.7, 1] & [0.6, 0.9] \\ [0.1, 0.4] & [0.4, 0.6] \\ [0.5, 0.6] & [0.3, 0.6] \\ [0.2, 0.4] & [0.8, 0.1] \end{bmatrix} \end{matrix}$$

and

$$R_2 = \begin{matrix} & d_1 & d_2 \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} & \begin{bmatrix} [0.0, 0.3] & [0.1, 0.4] \\ [0.6, 0.9] & [0.4, 0.6] \\ [0.4, 0.5] & [0.4, 0.7] \\ [0.6, 0.8] & [0.0, 0.2] \end{bmatrix} \end{matrix}$$

By our representation (2.1) we have,

$$R_{1L} = \begin{matrix} & d_1 & d_2 \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} & \begin{bmatrix} .7 & .6 \\ .1 & .4 \\ .5 & .3 \\ .2 & .8 \end{bmatrix} \end{matrix} \& R_{1U} = \begin{matrix} & d_1 & d_2 \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} & \begin{bmatrix} .1 & .9 \\ .4 & .6 \\ .6 & .6 \\ .4 & .1 \end{bmatrix} \end{matrix}$$

$$R_{2L} = \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} \begin{pmatrix} .0 & .1 \\ .6 & .4 \\ .4 & .4 \\ .6 & .0 \end{pmatrix} \quad \& \quad R_{2U} = \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} \begin{pmatrix} .3 & .4 \\ .9 & .6 \\ .5 & .7 \\ .8 & .2 \end{pmatrix}$$

Again we take $C=\{C_1, C_2, C_3\}$ as the universal set where C_1 , C_2 and C_3 represent cultivation respectively and $S=\{e_1, e_2, e_3, e_4\}$ as the set of parameters suppose that,
 $F_1(e_1) = \{<C_1, [.6, .9]>, <C_2, [.3, .5]>, <C_3, [.6, .8]>\}$
 $F_2(e_2) = \{<C_1, [.3, .5]>, <C_2, [.3, .7]>, <C_3, [.2, .6]>\}$
 $F_3(e_3) = \{<C_1, [.8, .1]>, <C_2, [.2, .4]>, <C_3, [.5, .7]>\}$ and
 $F_4(e_4) = \{<C_1, [.6, .9]>, <C_2, [.3, .5]>, <C_3, [.2, .5]>\}.$

The Interval valued fuzzy matrix (F_1, S) is another parameterized family of all interval valued fuzzy matrices and gives a collection of approximate description of the cultivation – plant growth in the Agriculture. This interval valued fuzzy matrix (F_1, S) represents a relation matrix Q called cultivation plant growth matrix given by,

$$Q = \begin{matrix} & e_1 & e_2 & e_3 & e_4 \end{matrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{pmatrix} [.6, .9] & [.3, .5] & [.8, .1] & [.6, .9] \\ [.3, .5] & [.3, .7] & [.2, .4] & [.3, .5] \\ [.6, .8] & [.2, .6] & [.5, .7] & [.2, .5] \end{pmatrix}$$

By our representation (2.1) we have, $Q = [Q_L, Q_U]$

$$Q_L = \begin{matrix} & e_1 & e_2 & e_3 & e_4 \end{matrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{pmatrix} .6 & .3 & .8 & .6 \\ .6 & .3 & .2 & .3 \\ .6 & .2 & .5 & .2 \end{pmatrix} \quad \text{and}$$

$$Q_U = \begin{matrix} & e_1 & e_2 & e_3 & e_4 \end{matrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{pmatrix} .9 & .5 & 1 & .9 \\ .5 & .7 & .4 & .5 \\ .8 & .6 & .7 & .5 \end{pmatrix}$$

Then combining the relation matrices R_{1L} , R_{1U} and R_{2L} , R_{2U} Separately with Q_L and Q_U we get the matrices $T_1 = [T_{1L}, T_{1U}]$ and $T_2 = [T_{2L}, T_{2U}]$ From equations (3.3), (3.4), (3.5) and (3.6) we have,

$$T_{1L} = \begin{matrix} & d_1 & d_2 \end{matrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{pmatrix} .6 & .6 \\ .3 & .3 \\ .6 & .6 \end{pmatrix} \quad \& \quad T_{1U} = \begin{matrix} & d_1 & d_2 \end{matrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{pmatrix} .9 & .9 \\ .5 & .6 \\ .8 & .8 \end{pmatrix}$$

$$T_{2L} = \begin{matrix} & d_1 & d_2 \end{matrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{pmatrix} .6 & .4 \\ .3 & .3 \\ .4 & .4 \end{pmatrix} \quad \& \quad T_{2U} = \begin{matrix} & d_1 & d_2 \end{matrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{pmatrix} .8 & .7 \\ .7 & .6 \\ .6 & .7 \end{pmatrix}$$

$$T_{3L} = \begin{matrix} & d_1 & d_2 \end{matrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{pmatrix} .6 & .7 \\ .3 & .3 \\ .5 & .5 \end{pmatrix} \quad \& \quad T_{3U} = \begin{matrix} & d_1 & d_2 \end{matrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{pmatrix} .6 & .4 \\ .6 & .4 \\ .6 & .4 \end{pmatrix}$$

$$T_{4L} = \begin{matrix} & d_1 & d_2 \end{matrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{pmatrix} .6 & .6 \\ .3 & .3 \\ .6 & .6 \end{pmatrix} \quad \& \quad T_{4U} = \begin{matrix} & d_1 & d_2 \end{matrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{pmatrix} .7 & .8 \\ .5 & .5 \\ .7 & .6 \end{pmatrix}$$

Now, from equations (3.7) and (3.8) we have,

$$S_{T1L} = \begin{matrix} & d_1 & d_2 \end{matrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{pmatrix} .6 & .6 \\ .3 & .3 \\ .6 & .6 \end{pmatrix} \quad \& \quad S_{T1U} = \begin{matrix} & d_1 & d_2 \end{matrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{pmatrix} .9 & .9 \\ .5 & .6 \\ .8 & .8 \end{pmatrix}$$

$$S_{T2L} = \begin{matrix} & d_1 & d_2 \end{matrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{pmatrix} .6 & .7 \\ .3 & .3 \\ .5 & .5 \end{pmatrix} \quad \& \quad S_{T2U} = \begin{matrix} & d_1 & d_2 \end{matrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{pmatrix} .8 & .7 \\ .7 & .6 \\ .6 & .7 \end{pmatrix}$$

We calculate the score for and against the diseases S_{T1} and S_{T2} from equations (3.9) and (3.10) we have,

$$S_{T1} = \begin{matrix} & d_1 & d_2 \end{matrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{pmatrix} .9 & .9 \\ .5 & .6 \\ .8 & .8 \end{pmatrix} \quad \& \quad S_{T2} = \begin{matrix} & d_1 & d_2 \end{matrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} \begin{pmatrix} .8 & .7 \\ .7 & .6 \\ .6 & .7 \end{pmatrix}$$

Now, for equation (3.11) we have the difference for and against the diseases by IVFM method.

Table 1

$S_{T1} \sim S_{T2}$	d_1	d_2
C_1	.1	.2
C_2	.2	.0

We conclude that cultivation C_3 is affected from the disease d_1 and cultivations C_1 and C_2 both affected from disease d_2 . From the above table 1, we conclude that Sanchez's approach can be applied to study the various diseases in plants by using the representation of an interval valued fuzzy matrix as an interval matrix of two fuzzy matrices.

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