Some New Classes of 3-Total Product Cordial Graphs

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Abstract - In [3], Ponraj R et al have defined the 3- Total Product Cordial of a graph G (V, E) as follows. Let f be a function from V(G) to {0, 1, ... k - 1} where k is an integer, 2 ≤ k ≤ |V(G)|. For each edge uv assign the label f(u) f(v) (mod k). f is called a k - Total Product cordial labeling if |f(i) - f(j)| ≤ 1, i, j ∈ {0, 1, ... k - 1} where f(x) denotes the total number of vertices and edges labeled with x (x = 0, 1, 2,..., k-1). We prove that the 3-Total Product cordial labeling is a behaviour of F_n.

Keywords- Binary labeling, cordial labeling, Product cordial, k-Total product cordial.

I. INTRODUCTION


A. Definition

The vertex labeling f * is said to be a binary labeling if f * : V(G) → {0, 1} such that each edge xy is assigned the label |f *(x) - f *(y)|.

B. Definition

A binary vertex labeling of a graph G is called a cordial labeling if |v_0| - |v_1| ≤ 1 and |e_0| - |e_1| ≤ 1. A graph G is called cordial if it admits cordial labeling.

C. Definition

A binary vertex labeling of graph G with induced edge labeling f*: E(G) → {0, 1} defined by f*(e = uv) = f(u)f(v) is called a Product cordial labeling if |v_i| - |v_{i+1}| ≤ 1 and |e_i| - |e_{i+1}| ≤ 1. A graph is called Product cordial if it admits Product cordial labeling.

D. Definition

A total product cordial labeling of a graph G is a function f : (V(G) U E(G)) → {0, 1} such that f(xy) = f(x)f(y) where x, y ∈ V(G), xy ∈ E(G) and the total number of 0’s and 1’s are balanced i.e. if v_i and e_i denote the set of vertices and edges which are labeled as i for i = 0, 1 respectively, then |v_i - v_{i+1}| ≤ 1. If there exists a total product cordial labeling of a graph G then it is called a Total product cordial graph.

E. Definition

Let f be a function from V(G) to {0, 1, ... k-1} where k is an integer, 2 ≤ k ≤ |V(G)|. For each edge uv assign the label f(u)f(v)(mod k). f is called a k-Total product cordial labeling if |f(i) - f(j)| ≤ 1, i, j ∈ {0, 1, ... k-1} where f(x) denotes the total number of vertices and edges labeled with x (x = 0, 1, 2,..., k-1). A graph that admits a k-Total product cordial labeling is called k-Total product cordial graph. M. Sundaram et al (2012) [3] proved the following graphs are total product cordial. Every product cordial graph of even order or odd order and even size; all cycles except C_n, C_n with m edges appended at each vertex; The fan graph F_{n-1} where (f_{n-1} = F_{n-1} + K_1) and wheel graph W_n where (W_n = C_{n-1} + K_1). The Helm graph is H_n (obtained from a wheel graph by attaching a pendant edge at each vertex of the n-cycle). A wheel graph W_n with n vertices is defined to be the join of C_{n-1} + K_1 of an isolated vertex with the cycle of length n. The helm graph H_n is the graph obtained from a wheel graph by attaching a pendant edge at each vertex of the n-cycle. The flower graph is the graph obtained from the helm by attaching each pendant edge vertex to the centre vertex of the Wheel (W_n). In this paper, we compute the 3-Total product cordial labeling on flower graphs.

II. MAIN RESULT

In this section we prove that the flower graph F_n is 3-Total product cordial graphs. We prove this in the following theorem:

Theorem : The flower graph F_n is 3 - Total product cordial labeling.

Proof: Denote the central vertex of the flower graph, F_n as u. The vertex u is called the hub vertex of the flower graph. Denote the vertices in the cycle of the flower as u_1, u_2, u_3, ..., u_{n-1}, u_n in the clockwise direction. Denote the end-vertices of the flower as u_1, u_2, u_3, ..., V_n in the clockwise direction. Denote the edges incident with u as c_1, c_2, c_3, ..., c_{m-1}, c_m in the clockwise direction. Denote the edges of the cycle in the flower as e_1, e_2, e_3, ..., e_{m-1}, e_m in the clockwise direction. Denote the pendant edges of the flower as d_1, d_2, d_3, ..., d_{m-1}, d_m in the clockwise direction. Denote the edge which is attached to the center vertex and with the pendant vertex as a_1, a_2, a_3, ..., a_{m-1}, a_m again in the clockwise direction.

Figure 1: 3-Total product cordial of flower F_n
In order to compute the 3-Total product cordial for the flower graph, we first label the vertex of the flower $F_n$ and verify that these vertex labels are 3-Total product cordial as follows.

Let $f$ be a function from $V(G)$ to $\{0, 1, 2\}$. For edge $uv$ assign the label $f(u)f(v) \pmod{3}$. Now we calculate the 3-Total product cordial for the flower graph as follows.

**Case 1:** When $n \equiv 0 \pmod{3}$, If $t = 3$; $n = 9$ Here $f(0) = 6t + 1 = 19$, $f(1) = f(2) = 6t = 18$

![Figure:5 Total product cordial on Flower graph $F_9$](image)

From the equation (1) the total number of vertices and edges labeled 0 is calculated as

$$f(0) = 6t + 1 \quad (2)$$

From the equation (1) the total number of vertices and edges labeled 1 and 2 is calculated as

$$f(1) = f(2) = 6t \quad (3)$$

Therefore $f$ is 3-Total product cordial labeling.

**Case 2**

When $n \equiv 1 \pmod{3}$

In this case we take $n - 1 = 3t$; $t = 1, 2, 3, \ldots$

Define the vertex labels of flower $F_n$ as follows.

$$f(u) = 0$$

$$f(u_i) = 2; 1 \leq i \leq n$$

$$f(v_i) = 2; 1 \leq i \leq n$$

From the equation (4) the total number of vertices and edges labeled 0 is calculated as

$$f(0) = 6t + 3 \quad (5)$$

![Figure:6 Total product cordial on Flower graph $F_{13}$](image)

Case 2 : When $n \equiv 1 \pmod{3}$, If $t = 4$; $n = 13$, Here $f(0) = 6t + 3 = 27$, $f(1) = f(2) = 6t + 2 = 26$

From the equation (5) the total number of vertices and edges labeled 1 and 2 is calculated as

$$f(1) = f(2) = 6t + 2 \quad (6)$$

Therefore $f$ is 3-Total product cordial labeling.

**Case 3**

When $n \equiv 2 \pmod{3}$

In this case we take $n - 2 = 3t$; $t = 1, 2, 3, \ldots$

Define the vertex labels of flower $F_n$ as follows:

$$f(u) = 0$$

$$f(u_i) = 2; 1 \leq i \leq n$$

$$f(v_i) = 2; 1 \leq i \leq n$$

From the equation (1) the total number of vertices and edges labeled 0 is calculated as

$$f(0) = 6t + 1$$

Therefore $f$ is 3-Total product cordial labeling.

**A. Case 1**

When $n \equiv 0 \pmod{3}$

In this case we take $n = 3t$; $t = 1, 2, 3, \ldots$

Define the vertex labels of flower $F_n$ as follows.

$$f(u) = 0$$

$$f(u_i) = 2; 1 \leq i \leq n$$

$$f(v_i) = 2; 1 \leq i \leq n$$

(1)
\[ f(u_i) = 2 \; ; 1 \leq i \leq n \]
\[ f(v_i) = 2 \; ; 1 \leq i \leq n \]  \hspace{1cm} (7)

From the equation (7) the total number of vertices and edges labeled 0 is calculated as
\[ f(0) = 6t + 5 \]  \hspace{1cm} (8)

Figure: 7 Total product cordial on Flower graph \( F_{11} \)

From the equation (3.8) the total number of vertices and edges labeled 1 and 2 is calculated as
\[ f(1) = f(2) = 6t + 4 \]  \hspace{1cm} (9)

Hence the graph \( F_n \) is 3–Total product cordial labeling.

The illustrations for the various cases of the above theorem are given in the appendix. Further the sub-division of flower graph which admits 3 – total product cordial is been proved and Cartesian product of flower graph with a complete graph \( K_1 \) can be computed.

**REFERENCES**


