

Fuzzy Max-Min Composition in Quality Specifications of Multi-Agent System

T.Pathinathan¹, J. Jon Arockiaraj², M.Poonthentral³,

¹Department of Mathematics, Loyola College, Chennai

²Department of Mathematics, St. Joseph's college of Arts and Science, Cuddalore

Email: jonarockiaraj@gmail.com

Abstract - In this paper a new technique named Fuzzy Max-Min Composition is used to obtain prioritization of quality specifications that assist quality engineer in achieving the desired level of quality for Multi-agent systems. Intuitionistic fuzzy Set has been used to capture the uncertainties associated with stakeholder's recommendation.

Keywords- FuzzySet; Intuitionistic fuzzy Set (IFS), Intuitionistic Fuzzy Relation (IFR) Multi-agent system (MAS), Quality Criteria, Quality factor.

I. INTRODUCTION

A Multi-agent system is a loosely coupled network of software agents that can move throughout a network of agent aware computers [7] and interact to solve problems, which are beyond the individual capacities or knowledge of each problem solver, while software agent is a computer program that is situated in some environment. Agents are characterized as goal-oriented, situation aware and proactive as well as reactive [3]. MASs have been applied in a variety of domains, including monitoring complex chemical processes [6], maintaining cellular switching systems [8], servicing mobile manipulator robot [9], etc. MAS would have several quality-related specifications such as knowledgeability, persistence, availability, extensibility, collaboration [1][2][5][10] etc. Several aspects of quality may conflict with each other and may be difficult to achieve. In this paper the quality of MAS from the viewpoint of various stakeholders, like project manager, software engineer, user and maintainer as the opinions of stake holders regarding quality specifications are subjective; Intuitionistic fuzzy sets are being used to give an integrated view that satisfies each of the stakeholders involved. Intuitionistic Fuzzy Sets (IFSs) are one of the interesting and useful generalizations of fuzzy set theory introduced by Atanassov [12] having membership, non-membership and hesitation part. Fuzzy sets are IFS but the converse is not necessarily true. This paper proposes an application of IFS to software engineering domain by achieving consensus among stakeholders' opinions for quality specifications and hence prioritizes them to achieve desired MAS quality.

II. BRIEF INTRODUCTION TO IFS

In an Intuitionistic Fuzzy set, we must use any two functions from the triplet: membership function, non-membership function, and hesitation margin. In other words, the application of intuitionistic fuzzy sets instead of fuzzy sets means the introduction of another degree of freedom into a set description. IFSs are extensions of the standard fuzzy sets. All results which hold of fuzzy sets can be transformed here too. Also any research based on fuzzy sets can be described in terms of

IFS. IFSs have applications in various areas such as in medical diagnosis, chemistry, and in decision making in Medicine.

III. PRELIMINARIES

We give some basic definitions, which are used, in our next section.

A. Definition

Let a set E be fixed. An Intuitionistic fuzzy set IFS A in E is an object of the following form

$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ Where the functions

$\mu_A: E \rightarrow [0, 1]$ and $\nu_A: E \rightarrow [0, 1]$ define degree of membership and degree of non-membership of the element $x \in E$ to the set A , which is a subset of E , and for every $x \in E$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. The amount $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ called the hesitation part, which may cater to either membership value or non-membership value or both.

B. Definition

If A and B are two IFS of the set E ,

$A \subset B$ iff $(\forall x \in X) (\mu_A(x) \leq \mu_B(x) \text{ and } \nu_A(x) \geq \nu_B(x))$

$A \supset B$ iff $B \subset A$,

$A = B$ iff $(\forall x \in E), \mu_A(x) = \mu_B(x) \text{ and } \nu_A(x) = \nu_B(x)$

$A \cup B = \{ \langle x, \min \{ \mu_A(x), \mu_B(x) \}, \max \{ \nu_A(x), \nu_B(x) \} \rangle / x \in E \}$

$A \cap B = \{ \langle x, \max \{ \mu_A(x), \mu_B(x) \}, \min \{ \nu_A(x), \nu_B(x) \} \rangle / x \in E \}$

C. Definition

If A is an IFS of X , then the max min-max

Composition [17] of the IFR $R (X \rightarrow Y)$

With A is an IFS B of Y denoted by $B = R \circ A$, and is defined by the membership function.

$$\mu_{R \circ A}(y) = \bigvee_x [\mu_A(x) \wedge \mu_R(x, y)]$$

And the non-membership function given by

$$\nu_{R \circ A}(y) = \bigwedge_x [\nu_A(x) \vee \nu_R(x, y)] \text{ (Here } \vee = \max, \wedge = \min)$$

D. Definition

Let $Q (X \rightarrow Y)$ and $R (Y \rightarrow Z)$ be two IFRs. The max-min-max composition $R \circ Q$ is the intuitionistic fuzzy relation from X to Z defined by the membership function $\mu_{R \circ Q}(x, z) = \bigvee [\mu_Q(x, y) \wedge \mu_R(y, z)]$ and the non-membership function given by $\nu_{R \circ Q}(x, z) = \bigwedge [\nu_Q(x, y) \vee \nu_R(y, z)] \forall (x, z) \in X \times Z$ and $\forall y \in Y$

IV. PRIORITIZING QUALITY SPECIFICATIONS OF MAS

In this section presents an application of intuitionistic fuzzy sets to quality model of MAS [15] that address various dimensions of quality in terms of factors and criteria. Criteria are stakeholders' view of quality. IFSs have been employed to capture the subjectiveness associated with quality criteria specified by various stakeholders. In a Software Engineering environment, suppose

C = a set of quality criteria;

F = a set of quality factors;

S = a set of stakeholders involved;

Determination of prioritized quality specifications for MAS involves mainly the following steps:

1. Determination of quality criteria C , quality factors F and stakeholders S in MAS.
2. Formulation of intuitionistic fuzzy relation Q between criteria C and stake holders S .
3. Formulation of intuitionistic fuzzy relation R between factors F and criteria C .
4. Determination of intuitionistic fuzzy relation $T = R \circ Q$, consisting of values for quality factors corresponding to various stakeholders obtained from composition of Intuitionistic Fuzzy Relations R and Q .

Through an IFR R from C to F which is assumed to be given by a quality engineer/a team of quality engineers who is/are able to translate their own perception of association and non-association respectively between criteria and factors. This concept can be extended to a finite number of stakeholders. Let there be n stakeholders $s_i, i = 1, 2, n$ in software project. Thus $s_i \in S$. Let R be an IFR($C \rightarrow F$) and construct an IFR Q from the set of stakeholders S to the set of criteria C . Clearly, the composition T of IFRs R and Q ($T = R \circ Q$) describes the state of the stakeholder in terms of the factors as an IFR from S to F given by the membership function $\mu_T(s_i, f) = \bigvee_{c \in C} [\mu_Q(s_i, c) \wedge \mu_R(c, f)] \forall c \in C$ and the non-membership function given by $\nu_T(s_i, f) = \bigwedge_{c \in C} [\nu_Q(s_i, c) \vee \nu_R(c, f)] \forall s_i \in S$ and $f \in F$. For a given R and Q , the relation $T = R \circ Q$ can be computed.

Algorithm

[1] Compute $T = R \circ Q$

[2] Compute W ,

{Where $W = \{\mu_T(s_i, f_j), \mu_T^c(s_i, f_j)\}$ non-members in T is $\mu_T^c(s_i, f_j) = 1 - \nu_T(s_i, f_j)$ } converting as members in W .

[3] Find Min $\{\mu_T(s_i, f_j), \mu_T^c(s_i, f_j)\}$

[4] Find Max $\{\text{Min}\{\mu_T(s_i, f_j), \mu_T^c(s_i, f_j)\}\}$

then we conclude that the stakeholders s_i are highly qualify from the quality factors f_j (i.e., $j = 1, 2, 3, 4, 5$ and $i = 1, 2, 3, 4$)

V. CASE STUDY

Five quality criteria namely communicative richness, decisiveness, goal driven, machine independence and average response time and five quality factors such as Collaboration, Knowledgeability, Performance, Persistence and Extensibility were selected to simplify the study. The stakeholders were asked to give their recommendation about quality criteria using Intuitionistic Fuzzy sets that Capture the recommendation in form of a (μ, ν) where μ represents the stakeholders who voted in favor of quality criteria; ν represent the stakeholders who did not vote for quality criteria Table 1 present relation Q regarding opinion of stakeholders to quality criteria using IFS Table 2 presents the relation R regarding opinion of quality expert regarding association of quality criteria to quality factors using IFS.

Table 3 obtains the relation T containing Stakeholders opinion in terms of quality factors using Table 1 and Table 2. Let there are five stake holders project manager, software engineer, user, maintainer i.e., $S = \{\text{stake holders, project manager, software engineer, user, maintainer}\}$ and a set of quality criteria $C = \{\text{Communication richness, Decisiveness, Goal driven, Machine Independence, Average response time}\}$. The intuitionistic fuzzy relation Q ($S \rightarrow C$) is given as in Table 1. Let the set of quality factors, $F = \{\text{collaboration, Knowledge ability, Performance, Persistence, Extensibility}\}$. The intuitionistic fuzzy relation R ($C \rightarrow F$) is given as in Table 1 and Table 2.

Table-1

Q	Communication richness	Decisiveness	Goal driven	Machine Independence	Average response time
Project manager	(0.4,0.2)	(0.5,0.2)	(0.8,0.1)	(0.2,0.6)	(0.8,0.1)
Software engineer	(0.8,0.1)	(0.8,0.2)	(0.8,0.1)	(0.4,0.3)	(0.6,0.2)
User	(0.2,0.2)	(0.5,0.3)	(0.9,0.0)	(0.2,0.7)	(0.8,0.1)
Maintainer	(0.4,0.4)	(0.7,0.1)	(0.9,0.1)	(0.8,0.1)	(0.4,0.3)

Table 2

R	Collaboration	Knowledge ability	Performance	Persistence	Extensibility
Communication richness	(0.7,0.2)	(0.8,0.1)	(0.6,0.3)	(0.4,0.3)	(0.2,0.6)
Decisiveness	(0.5,0.2)	(0.6,0.1)	(0.5,0.2)	(0.3,0.3)	(0.1,0.8)
Goal driven	(0.4,0.3)	(0.5,0.2)	(0.2,0.2)	(0.1,0.3)	(0.2,0.2)
Machine Independence	(0.1,0.5)	(0.1,0.6)	(0.3,0.2)	(0.2,0.4)	(0.7,0.2)
Average response time	(0.5,0.2)	(0.4,0.2)	(0.8,0.0)	(0.2,0.2)	(0.3,0.4)

Table 3

T	Collaboration	Knowledge ability	Performance	Persistence	Extensibility
Project Manager	(0.5,0.2)	(0.5,0.2)	(0.8,0.1)	(0.4,0.2)	(0.3, 0.2)
Software Engineer	(0.7,0.2)	(0.8,0.1)	(0.6,0.2)	(0.4,0.2)	(0.4,0.2)
User	(0.5,0.2)	(0.5,0.2)	(0.8,0.1)	(0.3,0.2)	(0.3,0.2)
Main Tainer	(0.5,0.2)	(0.6,0.1)	(0.5,0.2)	(0.4,0.3)	(0.7,0.2)

Table 4

W	Collaboration	Knowledge ability	Performance	Persistence	Extensibility
Project manager	(0.5,0.8)	(0.5,0.8)	(0.8,0.9)	(0.4,0.8)	(0.3,0.8)
Software engineer	(0.7,0.8)	(0.8,0.9)	(0.6,0.8)	(0.4,0.8)	(0.4,0.8)
User	(0.5,0.8)	(0.5,0.8)	(0.8,0.9)	(0.3,0.8)	(0.3,0.8)
Maintainer	(0.5,0.8)	(0.6,0.9)	(0.5,0.8)	(0.4,0.3)	(0.7,0.8)

Table 5

M	Collaboration	Knowledge ability	Performance	Persistence	Extensibility
Project manager	0.5	0.5	0.8	0.4	0.3
Software engineer	0.7	0.8	0.6	0.4	0.4
User	0.5	0.5	0.8	0.3	0.3
Maintainer	0.5	0.6	0.5	0.3	0.7

VI. CONCLUSION

From Table5 it is obvious that the Project Manager and user are highly qualified in Performance whereas Software engineer in knowledgeable and Maintainer in Extensibility

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CALCULATIONS

A. Calculations for Table 3

$$\mu_T(s_1, f_1) = \max(\min(0.4, 0.7), \min(0.5, 0.5), \min(0.8, 0.4), \min(0.2, 0.1), \min(0.8, 0.5)) = \max(0.4, 0.5, 0.4, 0.1, 0.5) = 0.5$$

$$v_T(s_1, f_1) = \min(\max(0.2, 0.2), \max(0.2, 0.2), \max(0.1, 0.3), \max(0.6, 0.5), \max(0.1, 0.2)) = \min(0.2, 0.2, 0.3, 0.6, 0.2) = 0.2$$

$$(\mu_T(s_1, f_1), v_T(s_1, f_1)) = (0.5, 0.2)$$

$$\mu_T(s_1, f_2) = \max(\min(0.4, 0.8), \min(0.5, 0.6), \min(0.8, 0.5), \min(0.2, 0.1), \min(0.8, 0.4)) = \max(0.4, 0.5, 0.5, 0.1, 0.4) = 0.5$$

$$v_T(s_1, f_2) = \min(\max(0.2, 0.1), \max(0.2, 0.1), \max(0.1, 0.2), \max(0.6, 0.6), \max(0.1, 0.2)) = \min(0.2, 0.2, 0.2, 0.6, 0.2) = 0.2$$

$$(\mu_T(s_1, f_2), v_T(s_1, f_2)) = (0.5, 0.2)$$

$$\mu_T(s_1, f_3) = \max(\min(0.4, 0.6), \min(0.5, 0.5), \min(0.8, 0.2), \min(0.2, 0.3), \min(0.8, 0.8)) = \max(0.4, 0.5, 0.2, 0.2, 0.8) = 0.8$$

$$v_T(s_1, f_3) = \min(\max(0.2, 0.3), \max(0.2, 0.2), \max(0.1, 0.2), \max(0.6, 0.2), \max(0.1, 0.0)) = \min(0.3, 0.2, 0.2, 0.6, 0.1) = 0.1$$

$$(\mu_T(s_1, f_3), v_T(s_1, f_3)) = (0.8, 0.1)$$

$$\mu_T(s_1, f_4) = \max(\min(0.4, 0.4), \min(0.5, 0.3), \min(0.8, 0.1), \min(0.2, 0.2), \min(0.8, 0.2)) = \max(0.4, 0.3, 0.1, 0.2, 0.2) = 0.4$$

$$v_T(s_1, f_4) = \min(\max(0.2, 0.3), \max(0.2, 0.3), \max(0.1, 0.3), \max(0.6, 0.4), \max(0.1, 0.2)) = \min(0.3, 0.3, 0.3, 0.6, 0.2) = 0.2$$

$$(\mu_T(s_1, f_4), v_T(s_1, f_4)) = (0.4, 0.2)$$

$$\mu_T(s_1, f_5) = \max(\min(0.4, 0.2), \min(0.5, 0.1), \min(0.8, 0.2), \min(0.2, 0.7), \min(0.8, 0.3)) = \max(0.2, 0.1, 0.2, 0.2, 0.3) = 0.3$$

$$v_T(s_1, f_5) = \min(\max(0.2, 0.6), \max(0.2, 0.8), \max(0.1, 0.2), \max(0.6, 0.2), \max(0.1, 0.4)) = \min(0.6, 0.8, 0.2, 0.6, 0.4) = 0.2$$

$$(\mu_T(s_1, f_5), v_T(s_1, f_5)) = (0.3, 0.2)$$

$$\mu_T(s_2, f_1) = \max(\min(0.8, 0.7), \min(0.8, 0.5), \min(0.8, 0.4), \min(0.4, 0.1), \min(0.6, 0.5)) = \max(0.7, 0.5, 0.4, 0.1, 0.5) = 0.7$$

$$v_T(s_2, f_1) = \min(\max(0.1, 0.2), \max(0.2, 0.2), \max(0.1, 0.3), \max(0.3, 0.5), \max(0.2, 0.2)) = \min(0.2, 0.2, 0.3, 0.5, 0.2) = 0.2$$

$$(\mu_T(s_2, f_1), v_T(s_2, f_1)) = (0.7, 0.2)$$

$$\mu_T(s_2, f_2) = \max(\min(0.8, 0.8), \min(0.8, 0.6), \min(0.8, 0.5), \min(0.4, 0.1), \min(0.6, 0.4)) = \max(0.8, 0.6, 0.5, 0.1, 0.4) = 0.8$$

$$v_T(s_2, f_2) = \min(\max(0.1, 0.1), \max(0.2, 0.1), \max(0.1, 0.2), \max(0.3, 0.6), \max(0.2, 0.2)) = \min(0.1, 0.2, 0.2, 0.6, 0.2) = 0.1$$

$$(\mu_T(s_2, f_2), v_T(s_2, f_2)) = (0.8, 0.1)$$

$$\mu_T(s_2, f_3) = \max(\min(0.8, 0.6), \min(0.8, 0.5), \min(0.8, 0.2), \min(0.4, 0.3), \min(0.6, 0.8)) = \max(0.6, 0.5, 0.2, 0.3, 0.6) = 0.6$$

$$v_T(s_2, f_3) = \min(\max(0.1, 0.3), \max(0.2, 0.2), \max(0.1, 0.2), \max(0.3, 0.2), \max(0.2, 0.0)) = \min(0.3, 0.2, 0.2, 0.3, 0.2) = 0.2$$

$$(\mu_T(s_2, f_3), v_T(s_2, f_3)) = (0.6, 0.2)$$

$$\mu_T(s_2, f_4) = \max(\min(0.8, 0.4), \min(0.8, 0.3), \min(0.8, 0.1), \min(0.4, 0.2), \min(0.6, 0.2)) = \max(0.4, 0.3, 0.1, 0.2, 0.2) = 0.4$$

$$v_T(s_2, f_4) = \min(\max(0.1, 0.3), \max(0.2, 0.3), \max(0.1, 0.3), \max(0.3, 0.4), \max(0.2, 0.2)) = \min(0.3, 0.3, 0.3, 0.4, 0.2) = 0.2$$

$$(\mu_T(s_2, f_4), v_T(s_2, f_4)) = (0.4, 0.2)$$

$$\mu_T(s_2, f_5) = \max(\min(0.8, 0.2), \min(0.8, 0.1), \min(0.8, 0.2), \min(0.4, 0.7), \min(0.6, 0.3)) = \max(0.2, 0.1, 0.2, 0.4, 0.3) = 0.4$$

$$v_T(s_2, f_5) = \min(\max(0.1, 0.6), \max(0.2, 0.8), \max(0.1, 0.2), \max(0.3, 0.2), \max(0.2, 0.4)) = \min(0.6, 0.8, 0.2, 0.3, 0.4) = 0.2$$

$$(\mu_T(s_2, f_5), v_T(s_2, f_5)) = (0.4, 0.2)$$

$$\mu_T(s_3, f_1) = \max(\min(0.2, 0.7), \min(0.5, 0.5), \min(0.9, 0.4), \min(0.2, 0.1), \min(0.8, 0.5)) = \max(0.2, 0.5, 0.4$$

$$, 0.1, 0.5) = 0.5$$

$$v_T(s_3, f_1) = \min(\max(0.2, 0.2), \max(0.3, 0.2), \max(0.0, 0.3), \max(0.7, 0.5), \max(0.1, 0.2)) = \min(0.2, 0.3, 0.3, 0.7, 0.2) = 0.2$$

$$(\mu_T(s_3, f_1), v_T(s_3, f_1)) = (0.5, 0.2)$$

$$\mu_T(s_3, f_2) = \max(\min(0.2, 0.8), \min(0.5, 0.6), \min(0.9, 0.5), \min(0.2, 0.1), \min(0.8, 0.4)) = \max(0.2, 0.5, 0.5, 0.1, 0.4) = 0.5$$

$$v_T(s_3, f_2) = \min(\max(0.2, 0.1), \max(0.3, 0.1), \max(0.0, 0.2), \max(0.7, 0.6), \max(0.1, 0.2)) = \min(0.2, 0.3, 0.2, 0.7, 0.2) = 0.2$$

$$(\mu_T(s_3, f_2), v_T(s_3, f_2)) = (0.5, 0.2)$$

$$\mu_T(s_3, f_3) = \max(\min(0.2, 0.6), \min(0.5, 0.5), \min(0.9, 0.2), \min(0.2, 0.3), \min(0.8, 0.8)) = \max(0.2, 0.5, 0.2, 0.2, 0.8) = 0.8$$

$$v_T(s_3, f_3) = \min(\max(0.2, 0.3), \max(0.3, 0.2), \max(0.0, 0.2), \max(0.7, 0.2), \max(0.1, 0.0)) = \min(0.2, 0.3, 0.2, 0.7, 0.1) = 0.1$$

$$(\mu_T(s_3, f_3), v_T(s_3, f_3)) = (0.8, 0.1)$$

$$\mu_T(s_3, f_4) = \max(\min(0.2, 0.4), \min(0.5, 0.3), \min(0.9, 0.1), \min(0.2, 0.2), \min(0.8, 0.2)) = \max(0.2, 0.3, 0.1, 0.2, 0.2) = 0.3$$

$$v_T(s_3, f_4) = \min(\max(0.2, 0.3), \max(0.3, 0.3), \max(0.0, 0.3), \max(0.7, 0.4), \max(0.1, 0.2)) = \min(0.3, 0.3, 0.3, 0.7, 0.2) = 0.2$$

$$(\mu_T(s_3, f_4), v_T(s_3, f_4)) = (0.3, 0.2)$$

$$\mu_T(s_3, f_5) = \max(\min(0.2, 0.7), \min(0.5, 0.1), \min(0.9, 0.2), \min(0.2, 0.7), \min(0.8, 0.3)) = \max(0.2, 0.1, 0.2, 0.2, 0.3) = 0.3$$

$$v_T(s_3, f_5) = \min(\max(0.2, 0.6), \max(0.3, 0.8), \max(0.0, 0.2), \max(0.7, 0.2), \max(0.1, 0.4)) = \min(0.6, 0.8, 0.2, 0.7, 0.4) = 0.2$$

$$(\mu_T(s_3, f_5), v_T(s_3, f_5)) = (0.3, 0.2)$$

$$\mu_T(s_4, f_1) = \max(\min(0.4, 0.7), \min(0.7, 0.5), \min(0.9, 0.4), \min(0.8, 0.1), \min(0.4, 0.5)) = \max(0.4, 0.5, 0.4, 0.1, 0.4) = 0.5$$

$$v_T(s_4, f_1) = \min(\max(0.2, 0.2), \max(0.1, 0.2),$$

$$\max(0.1, 0.3), \max(0.1, 0.5), \max(0.3, 0.2)) = \min(0.2, 0.2, 0.3, 0.5, 0.3) = 0.2$$

$$(\mu_T(s_4, f_1), \nu_T(s_4, f_1)) = (0.5, 0.2)$$

$$\mu_T(s_4, f_2) = \max(\min(0.4, 0.8), \min(0.7, 0.6), \min(0.9, 0.5), \min(0.8, 0.1), \min(0.4, 0.4)) = \max(0.4, 0.6, 0.5, 0.1, 0.4) = 0.6$$

$$\nu_T(s_4, f_2) = \min(\max(0.2, 0.1), \max(0.1, 0.1), \max(0.1, 0.2), \max(0.1, 0.6), \max(0.3, 0.2)) = \min(0.2, 0.1, 0.2, 0.6, 0.3) = 0.1$$

$$(\mu_T(s_4, f_2), \nu_T(s_4, f_2)) = (0.6, 0.1)$$

$$\mu_T(s_4, f_3) = \max(\min(0.4, 0.6), \min(0.7, 0.5), \min(0.9, 0.2), \min(0.8, 0.3), \min(0.4, 0.8)) = \max(0.4, 0.5, 0.2, 0.3, 0.4) = 0.5$$

$$\nu_T(s_4, f_3) = \min(\max(0.2, 0.3), \max(0.1, 0.2), \max(0.1, 0.2), \max(0.1, 0.2), \max(0.3, 0.0)) = \min(0.3, 0.2, 0.2, 0.2, 0.3) = 0.2$$

$$(\mu_T(s_4, f_3), \nu_T(s_4, f_3)) = (0.5, 0.2)$$

$$\mu_T(s_4, f_4) = \max(\min(0.4, 0.4), \min(0.7, 0.3), \min(0.9, 0.1), \min(0.8, 0.2), \min(0.4, 0.2)) = \max(0.4, 0.3, 0.1, 0.2, 0.2) = 0.4$$

$$\nu_T(s_4, f_4) = \min(\max(0.2, 0.3), \max(0.1, 0.3), \max(0.1, 0.3), \max(0.1, 0.4), \max(0.3, 0.2)) = \min(0.3, 0.3, 0.3, 0.4, 0.3) = 0.3$$

$$(\mu_T(s_4, f_4), \nu_T(s_4, f_4)) = (0.4, 0.3)$$

$$\mu_T(s_4, f_5) = \max(\min(0.4, 0.2), \min(0.7, 0.1), \min(0.9, 0.2), \min(0.8, 0.7), \min(0.4, 0.3)) = \max(0.2, 0.1, 0.2, 0.7, 0.3) = 0.7$$

$$\nu_T(s_4, f_5) = \min(\max(0.2, 0.6), \max(0.1, 0.8), \max(0.1, 0.2), \max(0.1, 0.2), \max(0.3, 0.4)) = \min(0.6, 0.8, 0.2, 0.2, 0.4) = 0.2$$

$$(\mu_T(s_4, f_5), \nu_T(s_4, f_5)) = (0.7, 0.2)$$

B. Calculation for Table 4

We compute $W = \{\mu_T(s_i, f_j), \mu_T^c(s_i, f_j)\}$ where $i=1, 2, 3, 4$ and $j=1, 2, 3, 4, 5$

For $i=1$ and $j=1$

$$\mu_T^c(s_1, f_1) = 1 - \nu_T(s_1, f_1) = 1 - 0.2 = 0.8$$

$$(\mu_T(s_1, f_1), \mu_T^c(s_1, f_1)) = (0.5, 0.8)$$

For $i=1$ and $j=2$

$$\mu_T^c(s_1, f_2) = 1 - \nu_T(s_1, f_2) = 1 - 0.2 = 0.8$$

$$(\mu_T(s_1, f_2), \mu_T^c(s_1, f_2)) = (0.5, 0.8)$$

For $i=1$ and $j=3$

$$\mu_T^c(s_1, f_3) = 1 - \nu_T(s_1, f_3) = 1 - 0.1 = 0.9$$

$$(\mu_T(s_1, f_3), \mu_T^c(s_1, f_3)) = (0.8, 0.9)$$

For $i=1$ and $j=4$

$$\mu_T^c(s_1, f_4) = 1 - \nu_T(s_1, f_4) = 1 - 0.2 = 0.8$$

$$(\mu_T(s_1, f_4), \mu_T^c(s_1, f_4)) = (0.4, 0.8)$$

For $i=1$ and $j=5$

$$\mu_T^c(s_1, f_5) = 1 - \nu_T(s_1, f_5) = 1 - 0.2 = 0.8$$

$$(\mu_T(s_1, f_5), \mu_T^c(s_1, f_5)) = (0.3, 0.8)$$

For $i=2$ and $j=1$

$$\mu_T^c(s_2, f_1) = 1 - \nu_T(s_2, f_1) = 1 - 0.2 = 0.8$$

$$(\mu_T(s_2, f_1), \mu_T^c(s_2, f_1)) = (0.7, 0.8)$$

For $i=2$ and $j=2$

$$\mu_T^c(s_2, f_2) = 1 - \nu_T(s_2, f_2) = 1 - 0.1 = 0.9$$

$$(\mu_T(s_2, f_2), \mu_T^c(s_2, f_2)) = (0.8, 0.9)$$

For $i=2$ and $j=3$

$$\mu_T^c(s_2, f_3) = 1 - \nu_T(s_2, f_3) = 1 - 0.2 = 0.8$$

$$(\mu_T(s_2, f_3), \mu_T^c(s_2, f_3)) = (0.6, 0.8)$$

For $i=2$ and $j=4$

$$\mu_T^c(s_2, f_4) = 1 - \nu_T(s_2, f_4) = 1 - 0.2 = 0.8$$

$$(\mu_T(s_2, f_4), \mu_T^c(s_2, f_4)) = (0.4, 0.8)$$

For $i=2$ and $j=5$

$$\mu_T^c(s_2, f_5) = 1 - \nu_T(s_2, f_5) = 1 - 0.2 = 0.8$$

$$(\mu_T(s_2, f_5), \mu_T^c(s_2, f_5)) = (0.4, 0.8)$$

For $i=3$ and $j=1$

$$\mu_T^c(s_3, f_1) = 1 - \nu_T(s_3, f_1) = 1 - 0.2 = 0.8$$

$$(\mu_T(s_3, f_1), \mu_T^c(s_3, f_1)) = (0.5, 0.8)$$

For $i=3$ and $j=2$

$$\mu_T^c(s_3, f_2) = 1 - \nu_T(s_3, f_2) = 1 - 0.2 = 0.8$$

$$(\mu_T(s_3, f_2), \mu_T^c(s_3, f_2)) = (0.5, 0.8)$$

For $i=3$ and $j=3$

$$\mu_T^c(s_3, f_3) = 1 - \nu_T(s_3, f_3) = 1 - 0.1 = 0.9$$

$$(\mu_T(s_3, f_3), \mu_T^c(s_3, f_3)) = (0.8, 0.9)$$

For $i=3$ and $j=4$

$$\mu_T^c(s_3, f_4) = 1 - \nu_T(s_3, f_4) = 1 - 0.2 = 0.8$$

$$(\mu_T(s_3, f_4), \mu_T^c(s_3, f_4)) = (0.3, 0.8)$$

For $i=3$ and $j=5$

$$\mu_T^c(s_3, f_5) = 1 - \nu_T(s_3, f_5) = 1 - 0.2 = 0.8$$

$$(\mu_T(s_3, f_5), \mu_T^c(s_3, f_5)) = (0.3, 0.8)$$

For $i=4$ and $j=1$

$$\mu_T^c(s_4, f_1) = 1 - \nu_T(s_4, f_1) = 1 - 0.2 = 0.8$$

$$(\mu_T(s_4, f_1), \mu_T^c(s_4, f_1)) = (0.5, 0.8)$$

For $i=4$ and $j=2$

$$\mu_T^c(s_4, f_2) = 1 - \nu_T(s_4, f_2) = 1 - 0.1 = 0.9$$

$$(\mu_T(s_4, f_2), \mu_T^c(s_4, f_2)) = (0.6, 0.9)$$

For $i=4$ and $j=3$

$$\mu_T^c(s_4, f_3) = 1 - \nu_T(s_4, f_3) = 1 - 0.2 = 0.8$$

$$(\mu_T(s_4, f_3), \mu_T^c(s_4, f_3)) = (0.5, 0.8)$$

For $i=4$ and $j=4$

$$\mu_T^c(s_4, f_4) = 1 - \nu_T(s_4, f_4) = 1 - 0.3 = 0.7$$

$$(\mu_T(s_4, f_4), \mu_T^c(s_4, f_4)) = (0.4, 0.7)$$

For $i=4$ and $j=5$

$$\mu_T^c(s_4, f_5) = 1 - \nu_T(s_4, f_5) = 1 - 0.2 = 0.8$$

$$(\mu_T(s_4, f_5), \mu_T^c(s_4, f_5)) = (0.7, 0.8)$$

C. Calculation for Table 5

We calculate $\min\{\mu_T(s_i, f_j), \mu_T^c(s_i, f_j)\}$ from Table 4:

$$\mu_T^c(s_1, f_1) = 1 - \nu_T(s_1, f_1) = 1 - 0.2 = 0.8$$

$$\min(\mu_T(s_1, f_1), \mu_T^c(s_1, f_1)) = \min(0.5, 0.8) = 0.5$$

$$\mu_T^c(s_1, f_2) = 1 - \nu_T(s_1, f_2) = 1 - 0.2 = 0.8$$

$$\min(\mu_T(s_1, f_2), \mu_T^c(s_1, f_2)) = \min(0.5, 0.8) = 0.5$$

$$\mu_T^c(s_1, f_3) = 1 - \nu_T(s_1, f_3) = 1 - 0.1 = 0.9$$

$$\min(\mu_T(s_1, f_3), \mu_T^c(s_1, f_3)) = \min(0.8, 0.9) = 0.8$$

$$\mu_T^c(s_1, f_4) = 1 - \nu_T(s_1, f_4) = 1 - 0.2 = 0.8$$

$$\min(\mu_T(s_1, f_4), \mu_T^c(s_1, f_4)) = \min(0.4, 0.8) = 0.4$$

$$\mu_T^c(s_1, f_5) = 1 - \nu_T(s_1, f_5) = 1 - 0.2 = 0.8$$

$$\min(\mu_T(s_1, f_5), \mu_T^c(s_1, f_5)) = \min(0.3, 0.8) = 0.3$$

$$\mu_T^c(s_2, f_1) = 1 - \nu_T(s_2, f_1) = 1 - 0.2 = 0.8$$

$$\min(\mu_T(s_2, f_1), \mu_T^c(s_2, f_1)) = \min(0.7, 0.8) = 0.7$$

$$\mu_T^c(s_2, f_2) = 1 - \nu_T(s_2, f_2) = 1 - 0.1 = 0.9$$

$$\min(\mu_T(s_2, f_2), \mu_T^c(s_2, f_2)) = \min(0.8, 0.9) = 0.8$$

$$\mu_T^c(s_2, f_3) = 1 - \nu_T(s_2, f_3) = 1 - 0.2 = 0.8$$

$$\min(\mu_T(s_2, f_3), \mu_T^c(s_2, f_3)) = \min(0.6, 0.8) = 0.6$$

$$\mu_T^c(s_2, f_4) = 1 - \nu_T(s_2, f_4) = 1 - 0.2 = 0.8$$

$$\min(\mu_T(s_2, f_4), \mu_T^c(s_2, f_4)) = \min(0.4, 0.8) = 0.4$$

$$\mu_T^c(s_2, f_5) = 1 - \nu_T(s_2, f_5) = 1 - 0.2 = 0.8$$

$$\min(\mu_T(s_2, f_5), \mu_T^c(s_2, f_5)) = \min(0.4, 0.8) = 0.4$$

$$\mu_T^c(s_3, f_1) = 1 - \nu_T(s_3, f_1) = 1 - 0.2 = 0.8$$

$$\min(\mu_T(s_3, f_1), \mu_T^c(s_3, f_1)) = \min(0.5, 0.8) = 0.5$$

$$\begin{aligned}
\mu_T^c(s_3, f_2) &= 1 - v_T(s_3, f_2) = 1 - 0.2 = 0.8 \\
\text{Min}(\mu_T(s_3, f_2), \mu_T^c(s_3, f_2)) &= \min(0.5, 0.8) = 0.5 \\
\mu_T^c(s_3, f_3) &= 1 - v_T(s_3, f_3) = 1 - 0.1 = 0.9 \\
\text{Min}(\mu_T(s_3, f_3), \mu_T^c(s_3, f_3)) &= \min(0.8, 0.9) = 0.8 \\
\mu_T^c(s_3, f_4) &= 1 - v_T(s_3, f_4) = 1 - 0.2 = 0.8 \\
\text{Min}(\mu_T(s_3, f_4), \mu_T^c(s_3, f_4)) &= \min(0.3, 0.8) = 0.3 \\
\mu_T^c(s_3, f_5) &= 1 - v_T(s_3, f_5) = 1 - 0.2 = 0.8 \\
\text{Min}(\mu_T(s_3, f_5), \mu_T^c(s_3, f_5)) &= \min(0.3, 0.8) = 0.3 \\
\mu_T^c(s_4, f_1) &= 1 - v_T(s_4, f_1) = 1 - 0.2 = 0.8 \\
\text{Min}(\mu_T(s_4, f_1), \mu_T^c(s_4, f_1)) &= \min(0.5, 0.8) = 0.5 \\
\mu_T^c(s_4, f_2) &= 1 - v_T(s_4, f_2) = 1 - 0.1 = 0.9 \\
\text{Min}(\mu_T(s_4, f_2), \mu_T^c(s_4, f_2)) &= \min(0.6, 0.9) = 0.6 \\
\mu_T^c(s_4, f_3) &= 1 - v_T(s_4, f_3) = 1 - 0.2 = 0.8 \\
\text{Min}(\mu_T(s_4, f_3), \mu_T^c(s_4, f_3)) &= \min(0.5, 0.8) = 0.5 \\
\mu_T^c(s_4, f_4) &= 1 - v_T(s_4, f_4) = 1 - 0.3 = 0.7 \\
\text{Min}(\mu_T(s_4, f_4), \mu_T^c(s_4, f_4)) &= \min(0.4, 0.7) = 0.4 \\
\mu_T^c(s_4, f_5) &= 1 - v_T(s_4, f_5) = 1 - 0.2 = 0.8 \\
\text{Min}(\mu_T(s_4, f_5), \mu_T^c(s_4, f_5)) &= \min(0.7, 0.8) = 0.7
\end{aligned}$$

From this we calculate

$$\text{Max}\{\text{Min}\{\mu_T(s_i, f_j), \mu_T^c(s_i, f_j)\} \text{ for } i=1,2,3,4 \text{ and } j=1,2,3,4,5\}$$

For $i=1$ and $j=1, 2, 3, 4, 5$ we get,

$$\begin{aligned}
&\text{Max}\{\min(\mu_T(s_1, f_1), \mu_T^c(s_1, f_1)), \min(\mu_T(s_1, f_2), \mu_T^c(s_1, f_2)), \\
&\min(\mu_T(s_1, f_3), \mu_T^c(s_1, f_3)), \min(\mu_T(s_1, f_4), \mu_T^c(s_1, f_4)), \\
&\min(\mu_T(s_1, f_5), \mu_T^c(s_1, f_5)), \} \\
&\text{Max}(0.5, 0.5, 0.8, 0.4, 0.3) = 0.8
\end{aligned}$$

For $i=2$ and $j=1, 2, 3, 4, 5$ we get,

$$\begin{aligned}
&\text{Max}\{\min(\mu_T(s_2, f_1), \mu_T^c(s_2, f_1)), \min(\mu_T(s_2, f_2), \mu_T^c(s_2, f_2)), \\
&\min(\mu_T(s_2, f_3), \mu_T^c(s_2, f_3)), \min(\mu_T(s_2, f_4), \mu_T^c(s_2, f_4)), \\
&\min(\mu_T(s_2, f_5), \mu_T^c(s_2, f_5)), \} \\
&\text{Max}(0.7, 0.8, 0.6, 0.4, 0.4) = 0.8
\end{aligned}$$

For $i=3, j=1,2,3,4$ we get,

$$\begin{aligned}
&\text{Max}\{\min(\mu_T(s_3, f_1), \mu_T^c(s_3, f_1)), \min(\mu_T(s_3, f_2), \mu_T^c(s_3, f_2)), \\
&\text{Min}(\mu_T(s_3, f_3), \mu_T^c(s_3, f_3)), \\
&\text{Min}(\mu_T(s_3, f_4), \mu_T^c(s_3, f_4)), \\
&\text{Min}(\mu_T(s_3, f_5), \mu_T^c(s_3, f_5)), \} = 0.8
\end{aligned}$$

For $i=4, j=1,2,3,4$ we get,

$$\begin{aligned}
&\text{Max}\{\min(\mu_T(s_4, f_1), \mu_T^c(s_4, f_1)), \min(\mu_T(s_4, f_2), \mu_T^c(s_4, f_2)), \\
&\min(\mu_T(s_4, f_3), \mu_T^c(s_4, f_3)), \min(\mu_T(s_4, f_4), \mu_T^c(s_4, f_4)), \\
&\min(\mu_T(s_4, f_5), \mu_T^c(s_4, f_5)), \} \\
&\text{Max}(0.5, 0.6, 0.5, 0.4, 0.7) = 0.7
\end{aligned}$$

From this highest value we conclude the respective Stakeholder is highly qualified from the respective quality factor.