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Bloom Graph

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Abstract - Grid, cylinder and torus networks are immensely used Graph networks studied by specialists in dynamical systems and probability.In this paper, a new kind of graph, Bloom Graph [B(m,n)], have been proposed. It possessesa unique properly of being both regular and planar. We study some of its topological properties including its Hamiltonian property.

Keywords: Interconnection network, Bloom graph, Planarity, Hamiltonicity.

I. INTRODUCTION

Parallel computing systems have been developed to meet the increasing demands on computing powers. A bottleneck in parallel computing systems is the communication between processors. Therefore, the performance of interconnection networks is a critical issue in parallel computing. This has been a major driving force for the research of inter connection networks. The study of interconnection networks in parallel computing system includes the performance and cost issues.

Grid graphs are very common and there is an extremely large literature devoted to counting structures in them. A two-dimensional grid graph is an mxn graph $G_{m,n}$ that is the graph Cartesian product $P_m X P_n$ of path graphs on m and n vertices. A Cylinder graph is an mxn graph $C_{m,n}$ that is the graph Cartesian product $P_m X C_n$ of path graph on m vertices and cycle on n vertices. A Torus graph is an mxn graph $T_{m,n}$ that is the graph Cartesian product $C_m X C_n$ of Cycles on m and n vertices.

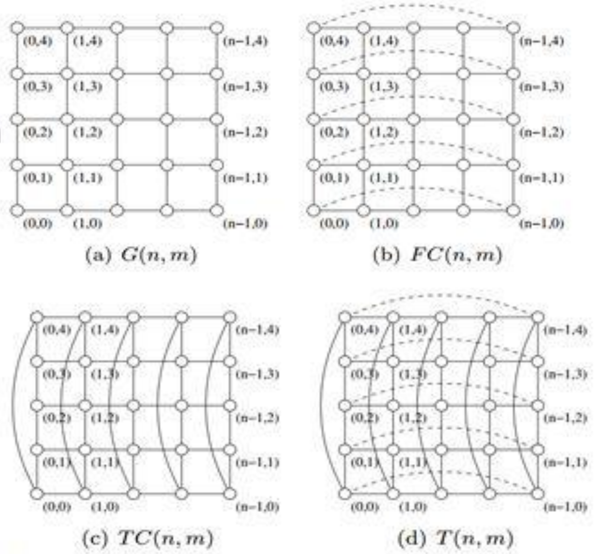


Fig 1. Grid, Cylinder and Torus

Grid and cylinder are planar but not regular. Whereas, torus is regular but not planar. In this paper, we introduce a new kind of graph which is both planar and regular.

II. BLOOM GRAPH

The Bloom Graph B(m,n); m,n>1 is defined as follows:
$V[B(m,n)] = \{(x,y) \mid 0 < x < m-1; 0 < y < n-1\}$, two distinct vertices (x_1, y_1) and (x_2, y_2) being adjacent if and only if

(i) $x_1 = x_2 - 1$ and $y_1 = y_2$
(ii) $x_1 = x_2 = 0$ and $y_1+1 \equiv y_2 \mod n$
(iii) $x_1 = x_2 = m$ and $y_1+1 \equiv y_2 \mod n$
(iv) $x_1 = x_2 - 1$ and $y_1 + 1 \equiv y_2 \mod n$

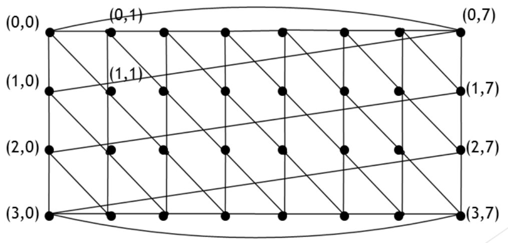
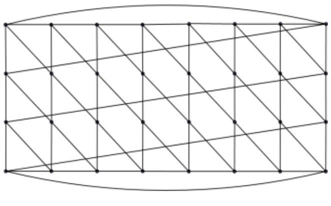


Fig 2. B(4,8)

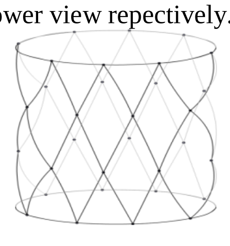
The condition (i) $x_1 = x_2 - 1$ and $y_1 = y_2$ describes the vertical edges. Condition (ii) $x_1 = x_2 = 0$ and $y_1+1 \equiv y_2 (\mod n)$ describes the horizontal edges in top most row. Condition (iii) $x_1 = x_2 = m$ and $y_1+1 \equiv y_2 (\mod n)$ describes the horizontal edges in lower most row. Condition (iv) $x_1 = x_2 - 1$ and $y_1 + 1 \equiv y_2 \mod n$ describes the slant edges.

III. PROPERTIES OF BLOOM GRAPH

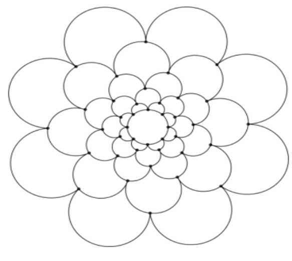
In order to understand the planarity of the bloom graph, it can redrawn as in fig 3 (b) and (c). The graphs in fig 3 (a) , (b) and (c) are isomporphic to each other giving a grid view, cylindrical view and a blooming flower view repectively.



(a)



(b)



(c)

Fig 3. Isomorphism of B(m,n)

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Theorem 1: Bloom Graph B(m,n) is planar.
Theorem 2: Bloom Graph B(m,n) is 4-regular.
Theorem 3: Bloom Graph B(m,n) have |V| = mn and |E| = 2mn.
Theorem 4: Bloom Graph B(m,n) have vertex connectivity 4 and edge connectivity 4.

IV. HAMILTONICITY

Theorem 5: Bloom Graph B(m,n) is hamiltonian.
Proof: The Hamiltonicity of B(m,n) can be discussed in two cases on the parity of m and n.

Case i: If m and n are of same parity. Then follow the Hamiltonian cycle as described in fig 4.

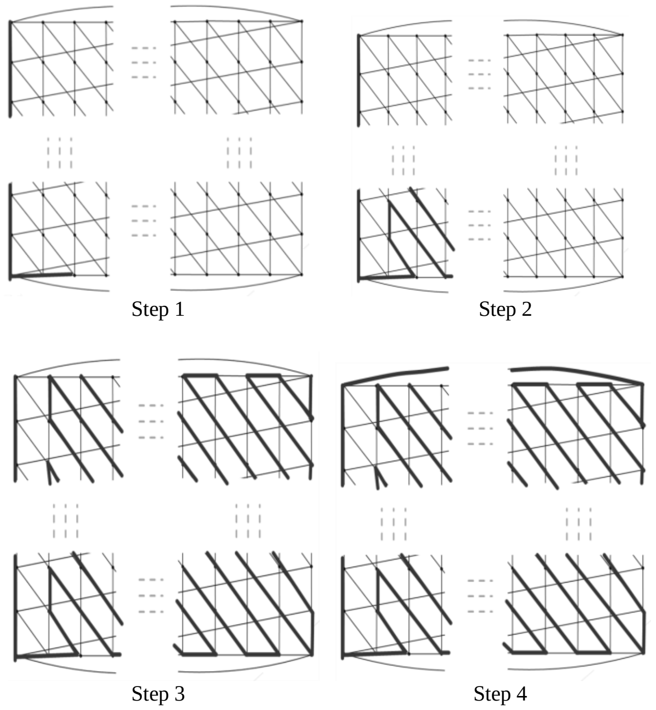

Fig 4.

Case ii: If m and n are of different parity. Then follow the Hamiltonian cycle as described in fig 5.

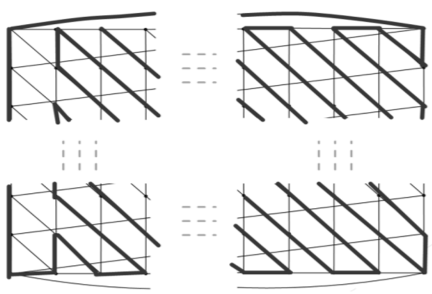

Fig 5.

V. CONCLUSION

In this paper we have considered a new interconnection network motivated by the grid, cylinder and torus network. The basic topological properties of Bloom graph have been discussed. The hamitonicity have also been proved. One can venture to explore the pancyclicity and broadcasting the proposed graph.

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