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b-Chromatic Number of Subdivision Edge and Vertex Corona

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Abstract - In this paper, we find that the b-chromatic number on corona graph of subdivision-vertex path with path. Then corona graph of any graph with path, cycle and complete graph and cycle with path.

Keywords: b-chromatic number, corona graph, subdivision-edge corona, subdivision-vertex corona, edge corona.

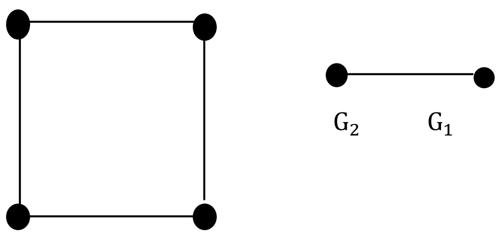
I. INTRODUCTION

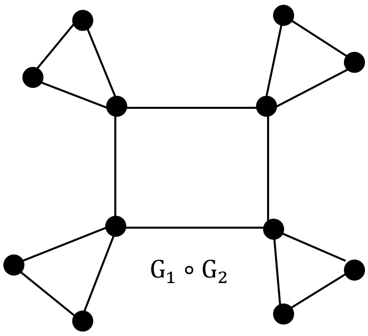
The b-chromatic number of a graph G, denoted by $\chi_b(G)$, is the maximal integer k such that G may have a b-coloring with k colors. This parameter has been defined by Irving and Manlove. The subdivision graph $S(G)$ of a graph G is the graph obtained by inserting a new vertex into every edge of G, we denote the set of such new vertices by $I(G)$.

In two new graph operations based on subdivision graphs, subdivision-vertex join and subdivision-edge join. The corona of two graphs was first introduced by R.Frucht and F.Harary in [11]. And another variant of the corona operation, the neighbourhood corona, was introduced in [12].

II. PRELIMINARIES

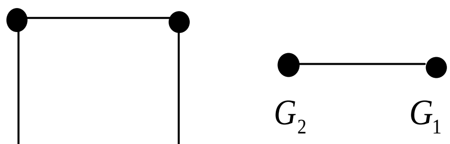
DEFINITION2.1.Let G_1 and G_2 be two graphs. Let $V(G_1)=\{v_1,v_2,...,v_k\}$ and take k copies of G_2. The corona $G_1 \circ G_2$ is the graph obtained by joining each V_i to every vertex of the i^{th} copy of G_2, $1 \le i \le k$.

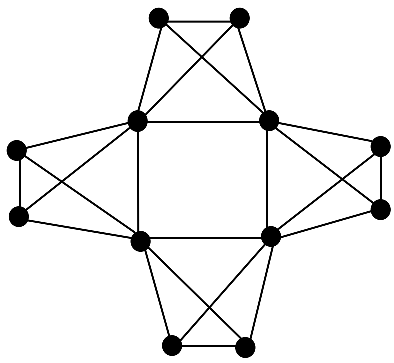




DEFINITION2.2. Let G_1 and G_2 be two graphs on disjoint sets of n_1 and n_2 vertices, m_1 and m_2 edges respectively. The edge corona $G_1 \diamond G_2$ of G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 and m_1 copies of G_2 and then joining two end-vertices of the i^{th} edge of G_1 to every vertex in the i^{th} copy of G_2.

Example: Let G_1 be the cycle of order 4 and G_2 be the complete graph k_2 of order 2. The two edge coronas $G_1 \diamond G_2$.





$G_1 \diamond G_2$

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III. SUBDIVISION-VERTEX AND SUBDIVISION-EDGE CORONA

DEFINITION 3.1. The Subdivision-vertex Corona of two vertex-disjoint graphs G_1 and G_2, denoted by $G_1 \odot G_2$ is the graph obtained from $S(G_1)$ and $|\mathrm{V}(\mathrm{G}_1)|$ copies of G_2, all vertex-disjoint by joining the i^{th} vertex of $\mathrm{V}(\mathrm{G}_1)$ to every vertex in the i^{th} copy of G_2.

DEFINITION3.2.The Subdivision-edge Corona of two vertex disjoint graphs G_1 and G_2 denoted by $G_1 \ominus G_2$ is the graph obtained from $S(G_1)$ and $|\mathrm{I}(\mathrm{G}_1)|$ copies of G_2, all vertex-disjoint, by joining the i^{th} vertex of $\mathrm{I}(\mathrm{G}_1)$ to every vertex in the i^{th} copy of G_2.

Let G_1 is a graph on n_1 vertices and m_1 edges and G_2 is a graph on n_2 vertices and m_2 edges then the subdivision-vertex Corona $G_1 \odot G_2$ has $n_1(1+n_2)+m_1$ vertices and $2m_1+n_1(n_2+m_2)$ edges, and the Subdivision-edge Corona $G_1 \ominus G_2$ has $m_1(1+n_2)+n_1$ vertices and $m_1(2+n_2+m_2)$ edges.

THEOREM 3.3. Let P_n be a path of n vertices and P_m be a path of m vertices. Then
$\chi_b(P_n \odot P_m) = \begin{cases} 2n-1 & n \geq m \\ m+1 & n < m \end{cases}$.

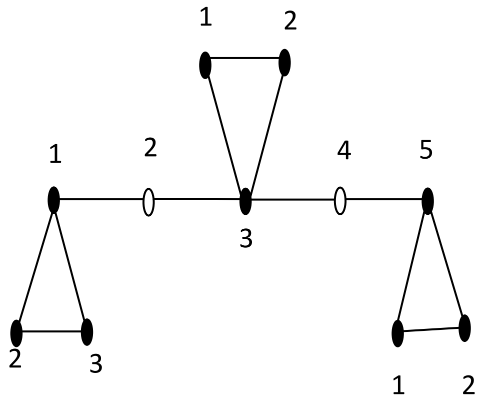
PROOF.
Let $V(P_n) = \{v_1, v_2, \dots, v_n\}$ and $V(P_m) = \{u_1, u_2, \dots, u_m\}$. let $V(P_n \circ P_m) = \{v_i : 1 \leq i \leq n\} \cup \{u_{ij} : 1 \leq i \leq m; 1 \leq j \leq m\}$.By the definition of corona graph, each vertex of P_n is adjacent to every vertex of a copy of P_m.

Assign the following n-coloring for $P_n \odot P_m$ as b-chromatic.

- For $1 \leq i \leq n$, assign the color c_i to v_i.
- For $1 \leq i \leq n$, assign the color c_i to u_{1i}, $\forall\, i \neq 1$.
- For $1 \leq i \leq n$, assign the color c_i to u_{2i}, $\forall\, i \neq 2$.
- For $1 \leq i \leq n$, assign the color c_i to u_{3i}, $\forall\, i \neq 3$.
- For $1 \leq i \leq n$, assign the color c_i to u_{ni}, $\forall\, i \neq n$.
- For $1 \leq i \leq n$, assign to vertex u_{ii} one of allowed colors.

Define $\chi_b(P_3 \odot P_2) = 5$.

- For $1 \leq i \leq 5$, assign the color c_i to v_i.
- For $1 \leq \ell \leq 5$, assign the color c_i to $u_{1i} \forall\, i \neq 1$.
- For $1 \leq \ell \leq 5$, assign the color c_i to $u_{2i} \forall\, i \neq 2$.
- For $1 \leq \ell \leq 5$, assign the color c_i to $u_{3i} \forall\, i \neq 3$.



$\chi_b(\mathrm{P}_3 \odot \mathrm{P}_2) = 5$

IV. VERTEX CORONA

4.1 GRAPHS WITH PATH

THEOREM4.1.1 Let Gbe a simple graph on n vertices. Then
$\chi_b(\mathrm{G} \circ \mathrm{P}_n) = \begin{cases} n+1, & for \quad n \leq 3 \\ n & for \quad n > 3 \end{cases}$

PROOF:
Let $\mathrm{V(G)} = \{v_1, v_2, \dots, v_n\}$ and $\mathrm{V(P}_n) = \{u_1, u_2, \dots, u_n\}$.
Let $\mathrm{V}(\mathrm{G} \circ P_n) = \{v_i : 1 \leq i \leq n\} \cup \{U_{ij} : 1 \leq i \leq n; 1 \leq j \leq n\}$.

By the definition of Corona graph each vertex of G is adjacent to every vertex of n copy of P_n. i.e) every vertex $v_i \in \mathrm{V}(G)$ is adjacent to every vertex from the set $\{u_{ij} : 1 \leq j \leq n\}$.

Assign the following n -coloring for $\mathrm{G} \circ P_n$ as b-chromatic

- For $1 \leq i \leq n$, assign the color c_i to v_i.
- For $1 \leq i \leq n$, assign the color c_i to u_{1i}, $\forall i \neq 1$..
- For $1 \leq i \leq n$, assign the color c_i to u_{2i}, $\forall i \neq 2$..
- For $1 \leq i \leq n$, assign the color v_i to u_{ni}, $\forall i \neq n$..
- For $1 \leq i \leq n$, assign the vertex u_{ii} one of allowed colors-such color exists because $2 \leq \deg(\mathrm{u}_{ii}) \leq 3$ and $n > 3$.

Let us assume that $\chi_b(\mathrm{G} \circ \mathrm{P}_n)$ is greater than n ie) $\chi_b(\mathrm{G} \circ \mathrm{P}_n) = n+1 \;\; \forall n > 3$ there must be at least $n+1$ vertices of degree n in $\mathrm{G} \circ \mathrm{P}_n$ all with distinct colors, and each adjacent to vertices of all of the other colors. But then these must be the vertices $\{v_1, v_2, \dots, v_n\}$. Since these are only

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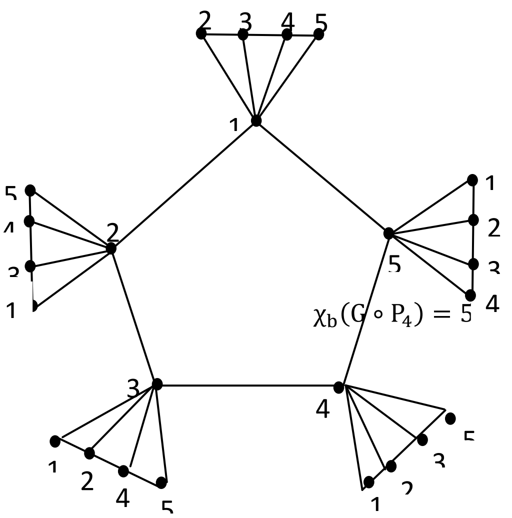
ones with degree at least n. This is a contradiction. b-coloring with $n+1$ colors is impossible.

Thus we have $\chi_b(G \circ P_n) \le n$. Hence $\chi_b(G \circ P_n) = n$, $\forall n > 3$.

Define b-coloring of $G \circ P_4(|V(G)|) = 4$ with 5 colors in the following way:

- For $1 \le i \le 5$, assign the color c_i to v_i.
- For $1 \le l \le 5$, assign the color c_l to u_{1l}, $\forall l \ne 1$.
- For $1 \le l \le 5$, assign the color c_l to u_{2l}, $\forall l \ne 2$.
- For $1 \le l \le 5$, assign the color c_l to u_{3l}, $\forall l \ne 3$.
- For $1 \le l \le 5$, assign the color c_l to u_{4l}, $\forall l \ne 4$.
- For $1 \le l \le 5$, assign the color c_l to u_{5l}, $\forall l \ne 5$.

We have
$\chi_b(G \circ P_4) = 5$. Hence $\chi_b(G \circ P_n) = n$ $\forall n > 3$.



4.2. GRAPH WITH CYCLE

THEOREM4.2.1. Let G be a simple graph on n vertices $n > 3$. Then $\chi_b(G \circ C_n) = n$.

Proof:
Let $V(G) = \{v_1, v_2, ..., v_n\}$ and $V(C_n) = \{u_1, u_2, ..., u_n\}$. Let $V(G \circ C_n) = \{v_i : 1 \le i \le n\} \cup \{u_{ij} : 1 \le i \le n; 1 \le j \le n\}$. By the definition of Corona graph, each vertex of G is adjacent to every vertex of a copy of C_n. i.e., every vertex $v_i \in V(G)$ is adjacent to every vertex from the set $\{u_{ij} : 1 \le j \le n\}$.

Assign the following n-coloring for $G \circ C_n$ as b-chromatic:

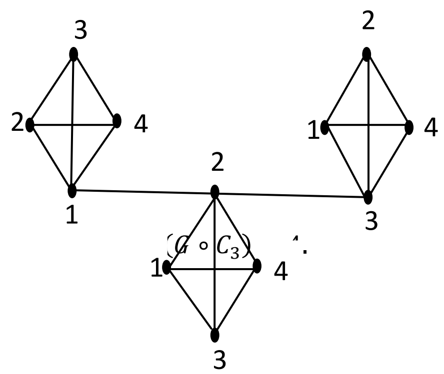
- For $1 \le i \le n$, assign the color c_i to v_i.
- For $1 \le i \le n$, assign the color c_i to u_{1i}, $\forall i \ne 1$.
- For $1 \le i \le n$, assign the color c_i to u_{2i}, $\forall i \ne 2$.
- For $1 \le i \le n$, assign the color c_i to u_{ni}, $\forall i \ne n$.
- For $1 \le i \le n$, assign to vertex u_{ii} one of allowed

colors-such color exists, because $\deg(u_{ii}) = 3$ and $n > 3$.

$\chi_b(G \circ C_n) \ge n$. Assume that $\chi_b(G \circ C_n)$ is greater than n, ie) $\chi_b(G \circ C_n) = n+1$ $\forall n > 3$, there must be at least $n+1$ vertices of degree n in $G \circ C_n$, all with distinct colors, and each adjacent to vertices of all of the other colors. But then these must be the vertices $\{v_1, v_2, ..., v_n\}$. Since these are only ones with degree at least n. This is the contradiction b-coloring with $n+1$ color is impossible.

We have $\chi_b(G \circ C_n) = n$. Hence $\chi_b(G \circ C_n) = n$, $\forall n > 3$.

$\chi_b(G \circ C_3) = 4$.



4.3. CYCLE WITH PATH

THEOREM 4.3.1Let C_n be a cycle of n vertices and P_n be a path of n vertices. Then $\chi_b(C_n \circ P_n) = n$

Proof:
Let $V(C_n) = \{v_1, v_2, ..., v_n\}$ and $V(P_n) = \{u_1, u_2, ..., u_n\}$ .Let $V(C_n \circ P_n) = \{v_i : 1 \le i \le n\} \cup \{u_{ij} : 1 \le i \le n; 1 \le j \le n\}$, By the definition of Corona graph, each vertex of G is adjacent to every vertex of a copy of P_n. i.e., every vertex $v_i \in V(C_n)$ is adjacent to every vertex from the set $\{u_{ij} : 1 \le j \le n\}$.

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$\chi_b(C_n \circ P_n) \geq n$. Assign the following b-coloring for $C_n \circ P_n$.

- For $1 \leq i \leq n$, assign the color c_i to v_i.
- For $1 \leq i \leq n$, assign the color c_i to u_{1i}, $\forall i \neq 1$.
- For $1 \leq i \leq n$, assign the color c_i to u_{2i}, $\forall i \neq 2$.

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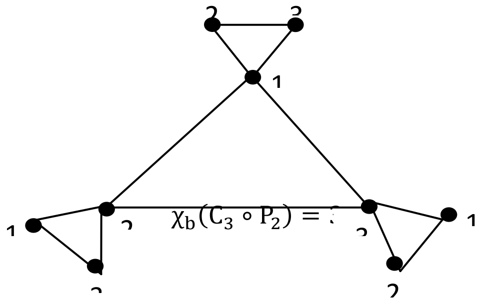
- For $1 \leq i \leq n$, assign the color c_i to u_{ni}, $\forall i \neq n$.
- For $1 \leq i \leq n$, assign to vertex u_{ii} one of allowed colors such color exists, because $2 \leq \deg(u_{ii}) \leq 3$ and $n \geq 3$.

We have $\chi_b(C_n \circ P_n) \leq n$. Hence $\chi_b(C_n \circ P_n) = n$.

We have $\chi_b(C_3 \circ P_2) = 3$.



4.4. GRAPH WITH COMPLETE GRAPH

Theorem4.4.1 :Let G be a simple graph on n vertices. Then $\chi_b(G \circ K_n) = n+1$.

Proof.

Let $V(G) = \{v_1, v_2, ..., v_n\}$ and $V(K_n) = \{u_1, u_2, ..., u_n\}$. Let $V(G \circ K_n) = \{v_i : 1 \leq i \leq n\} \cup \{u_{ij} : 1 \leq i \leq n; 1 \leq j \leq n\}$. By the definition of corona graph, each vertex of G is adjacent to every vertex of a copy of K_n. i.e., every vertex $v_i \in V(G)$ is adjacent to every vertex from the set $\{u_{ij} : 1 \leq j \leq n\}$.

Assign the following $n+1$-coloring for $G \circ K_n$ as b-chromatic:

- For $1 \leq i \leq n$, assign the color c_i to v_i.
- For $1 \leq l \leq n$, assign the color c_l to u_{1l}, $\forall l \neq 1$.
- For $1 \leq l \leq n$, assign the color c_l to u_{2l}, $\forall l \neq 2$.
- For $1 \leq l \leq n$, assign the color c_l to u_{3l}, $\forall l \neq 3$.
- For $1 \leq l \leq n$, assign the color c_l to u_{4l}, $\forall l \neq 4$.

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- For $1 \leq l \leq n$, assign the color c_l to u_{nl}, $\forall l \neq n$.
- For $1 \leq l \leq n$, assign the color c_{n+1} and u_{ll} Therefore, $\chi_b(G \circ K_n) \geq n+1$.

Let us assume that $\chi_b(G \circ K_n)$ is greater than $n+1$, i.e., $\chi_b(G \circ K_n) = n+2$, there must be at least $n+2$ vertices of degree $n+1$ in $G \circ K_n$, all with distinct colors, and each adjacent to vertices of all of the other colors. But then these must be the vertices $v_1, v_2, ...v_n$, since these are only ones with degree at least $n+1$. This is the contradiction, b-coloring with $n+2$ colors is impossible. Thus, we have $\chi_b(G \circ K_n) \leq n+1$.. Hence,.

V. CONCLUSION

In this existing subdivision-vertex corona graphs, they used spectrum. Here we study about subdivision- vertex corona graph using b-chromatic number.

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