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Applications of Mathematics in Circuit Theory

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Abstract - Application of Gaussian in circuit theory, using Kirchhoff's 2nd law. In this paper for a given circuit, forming into matrices form by using Kirchhoff's 2nd law we solve and find the current values. Less than 3x3 matrices we can use Carmel's rule, but more that 3x3, Carmel's cannot be done, so gauss elimination method is used to find the current values for the given circuits

I. CIRCUITS

An electronic circuit is composed of individual electronic components like Transistors, Capacitors, Inductors and Diodes, Resistors connected by conductive wires or traces through which Electric Current can flow. The combination of components and wires allows various simple and complex operations to be performed: signals can be amplified, computations can be performed, and data can be moved from one place to another. Circuits can be constructed of discrete components connected by individual pieces of wire

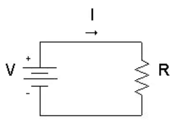
Let R = Resistance of the circuit
C = Capacitance in series with R
I = Current flowing
L = Inductor
V_R = voltage across R
Vc = voltage across C
V_L = voltage across L

II. OHM'S LAW

Ohm's law defines a linear relationship between the voltage and the current in an electrical circuit. The DC current flow through a resistor is set by the resistor's voltage drop and the resistor's resistance.

Ohm's Law Formula / Equation
When we know the voltage and resistance, we can calculate the current.

Ohm's law definition
The resistor's current I in amps (A) is equal to the resistor's voltage V_R =V in volts (V) divided by the resistance R in ohms (Ω):



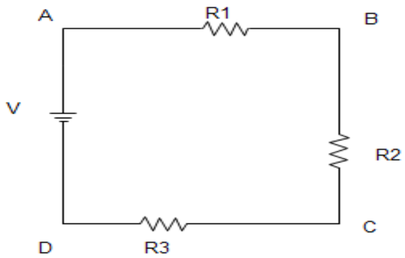
$$I = \frac{V}{R}$$

In 1845, a German physicist, Gustav Kirchhoff developed a pair or set of rules or laws which deal with the conservation of current and energy within electrical circuits. Application of Gauss Elimination in circuits

III. KIRCHHOFF ' S 2ND LAW

In a closed circuit the sum of the potential drops is equal to the sum of the potential rises.

In the closed loop ABCDA,

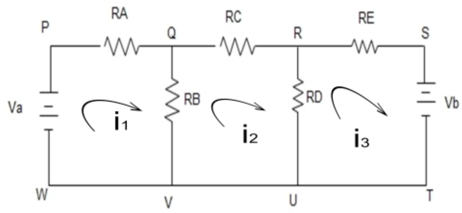


Branch	Potential drop	Potential rise
AB	IR_1	_
BC	IR_2	_
CD	IR_3	_
DA	_	V

Hence $IR_1 + IR_2 + IR_3 = V$

[Note: When we go from D to A (from the negative terminal to the positive terminal of the battery) There is a potential rise of V volts.

Consider the network shown below:



Assume the loop current to be I_1, I_2 and I_3 as shown in the figure, all clockwise.
The currents through R_A, R_C, R_E are I_1, I_2, I_3 respectively.
The current through R_B is $I_1 - I_2$ & through R_D is $I_2 - I_3$.

Branch	Potential rise	Potential drop
PQ	$I_1 R_A$	_
QV	$(I_1 - I_2) R_B$	_
VW	_	_
WP	_	V_a

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Hence $I_1 R_A + (I_1 - I_2) R_B = V_a$ $I_1(R_A + R_B) - I_2 R_B = V_a$ ……………….(1)

Similarly,

$I_2 R_C + (I_2 - I_3) R_D + (I_2 - I_1) R_B = 0$

$-I_1 R_B + (R_B + R_C + R_D) I_2 - I_3 R_D = 0$ ………..(2)

$I_3 R_E + (I_3 - I_2) R_D = -V_b$

$-I_2 R_D + (R_D + R_E) I_3 = -V_b$ …………(3)

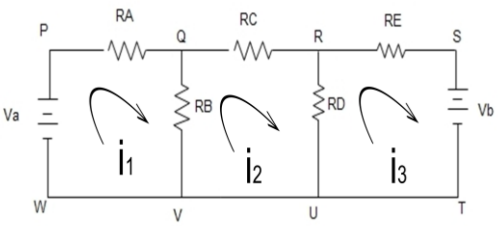
The resistance which is common to more than one loop is called mutual resistance

$R_B = R_{12} = R_{21}$
$R_D = R_{23} = R_{32}$

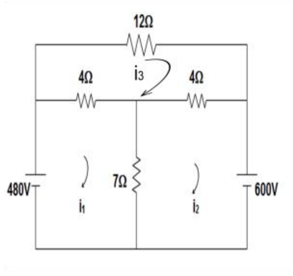
It is usual to write these Equations in terms of self and mutual resistances. The self resistance of a loop is the sum of the resistances encountered in a traverse of that loop.

Thus for the circuit drawn,

The self resistance of loop 1 : $R_{11} = R_A + R_B$
The self resistance of loop 2 : $R_{22} = R_B + R_C + R_D$
The self resistance of loop 3: $R_{33} = R_D + R_E$



Example: 3x3 matrix



$$\begin{pmatrix} 11 & -7 & -4 \\ -7 & 11 & -4 \\ -4 & -4 & 20 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 480 \\ -600 \\ 0 \end{pmatrix}$$

Eqns/. (1),(2) & (3)

$R_{11} I_1 - R_{12} I_2 = V_a$

$-R_{21} I_1 + R_{22} I_2 - R_{23} I_3 = 0$

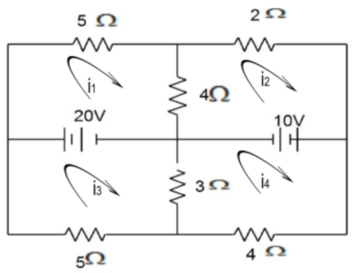
$-R_{32} I_2 + R_{33} I_3 = -V_b$

Find the Current value for the given circuits:

$$\Delta = \begin{vmatrix} 11 & -7 & -4 \\ -7 & 11 & -4 \\ -4 & -4 & 20 \end{vmatrix}$$

$= 11(204) + 7(-140-16) - 4(28+44)$
$=2244-1092-288$
$=864$

$\Delta_3 =$
$=480(72)+600(-44-28)$
$= -8640$
$I_3 = \Delta_3 / \Delta = -8640/864 = -10$ A



$$\begin{pmatrix} 9 & -4 & 0 & 0 \\ -4 & 6 & 0 & 0 \\ 0 & 0 & 8 & -3 \\ 0 & 0 & -3 & 7 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} 20 \\ -10 \\ -20 \\ 10 \end{pmatrix}$$

$$\begin{pmatrix} 9 & -4 & 0 & 0 & 20 \\ -4 & 6 & 0 & 0 & -10 \\ 0 & 0 & 8 & -3 & -20 \\ 0 & 0 & -3 & 7 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -4/9 & 0 & 0 & 20/9 \\ 0 & 38/9 & 0 & 0 & -10/9 \\ 0 & 0 & 8 & -3 & -20 \\ 0 & 0 & -3 & 7 & 10 \end{pmatrix} \quad R_1 \to R_1/9, R_2 \to R_2 + 4R_1$$

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$$\begin{pmatrix} 1 & -4/9 & 0 & 0 & 20/9 \\ 0 & 1 & 0 & 0 & -10/38 \\ 0 & 0 & 8 & -3 & -20 \\ 0 & 0 & -3 & 7 & 10 \end{pmatrix} R_2 \rightarrow R_2 \text{ X } \frac{9}{38}$$

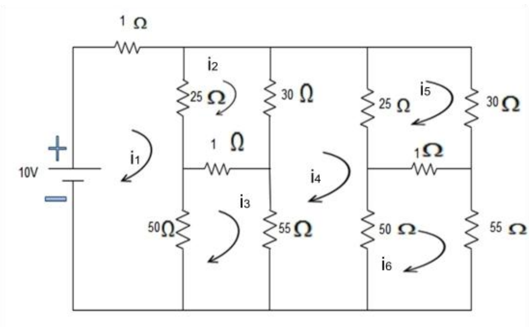
$$\begin{pmatrix} 1 & -4/9 & 0 & 0 & 20/9 \\ 0 & 1 & 0 & 0 & -10/38 \\ 0 & 0 & 1 & -3/8 & -20/8 \\ 0 & 0 & -3 & 7 & 10 \end{pmatrix} R_3 \rightarrow \frac{R_3}{8}$$

$$\begin{pmatrix} 1 & -4/9 & 0 & 0 & 20/9 \\ 0 & 1 & 0 & 0 & -10/38 \\ 0 & 0 & 1 & -3/8 & -20/8 \\ 0 & 0 & 0 & 47/8 & 20/8 \end{pmatrix} R_4 \rightarrow R_4 + 3R_3$$

$$\begin{pmatrix} 1 & -4/9 & 0 & 0 & 20/9 \\ 0 & 1 & 0 & 0 & -10/38 \\ 0 & 0 & 1 & -3/8 & -20/8 \\ 0 & 0 & 0 & 1 & 20/47 \end{pmatrix} R_4 \rightarrow R_4 \text{ X } \frac{8}{47}$$

CONCLUSION:

GAUSS ELIMINATION CIRCUIT IN 6X6 MATRIX



$$\begin{array}{c|c} & \begin{array}{cccccc} i1 & i2 & i3 & i4 & i5 & i6 \end{array} \\ \begin{array}{c} i1 \\ i2 \\ i3 \\ i4 \\ i5 \\ i6 \end{array} & \begin{pmatrix} 76 & -25 & -50 & 0 & 0 & 0 \\ -25 & 56 & -1 & -30 & 0 & 0 \\ -50 & -1 & 106 & -55 & 0 & 0 \\ 0 & -30 & -55 & 160 & -25 & -50 \\ 0 & 0 & 0 & -25 & 56 & -1 \\ 0 & 0 & 0 & -50 & -1 & 106 \end{pmatrix} \end{array} \begin{pmatrix} I1 \\ I2 \\ I3 \\ I4 \\ I5 \\ I6 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$i_4 = \frac{20}{47} \qquad \Rightarrow i_4 = 0.426\text{A}$$

$$i_3 + \frac{-3}{8} i_4 = \frac{-20}{8} \quad \Rightarrow i_3 = -2.340\text{A}$$

$$i_2 = \frac{-10}{38} \qquad \Rightarrow i_2 = -0.263\text{A}$$

$$i_1 + \frac{-4}{9} i_2 = \frac{20}{9} \quad \Rightarrow i_1 = 2.11 \text{ A}$$

$I_1 = 0.478$ A

$I_2 = 0.348$ A

$I_{3.} = 0.353$ A

$I_4 = 0.239$ A

$I_5 = 0.109$ A

$I_6 = 0.11\ 4$ A

we conclude that Application of Gaussian in circuit theory, using Kirchhoff's 2nd can be done for any type of matrices in order to find the current values for the given circuits . In this paper for a given circuit, forming into matrices form by using Kirchhoff's 2nd law we solve and find the current values. Less than 3x3 matrices we can use Carmel's rule, but more that 3x3, Carmel's cannot be done, so gauss elimination method is used to find the current values for the given circuits. Applying Kirchhoff's 2nd law we can find 6x6 matrices and more set of matrix form and find the current values as follows.

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