

An Application of Interval Valued Intuitionistic Fuzzy Sets in Medical Diagnosis Using Logical Operators

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Abstract- In this paper, it is proposed a new technique to diagnose the symptom of the disease using Interval Valued Intuitionistic fuzzy set (IVIFS) with logical operators. The membership and non-membership values are not always possible up to our satisfaction, but in deterministic (hesitation) part has more important role here, the fact that in decision making, particularly in case of medical diagnosis, there is a fair chance of the existence of a non-zero hesitation part at each moment of evaluation.

Keywords: Interval-Valued Intuitionistic Fuzzy Sets (IVIFS); Fuzzy logic (FL); Logical Operators; Medical diagnosis.

I. INTRODUCTION

In the real world, we frequently deal with vague or imprecise information. Information available is sometime vague, sometimes inexact or insufficient. Out of several higher order fuzzy sets, Intuitionistic fuzzy sets (IFS) (K. Atanassov, 1986; 1994) have been found to be highly useful to deal with vagueness. There are situations where due to insufficiency in the information available, the evaluation of member values is not possible up to our satisfaction. Due to the same reason, evaluations of non-membership values is not also always possible and consequently there remains a part in deterministic on which hesitation survives. Certainly fuzzy set theory is not appropriate to deal with such problem, rather IFS theory is more suitable. Out of several generalization of fuzzy set theory for various objectives, the notion introduced by Atanassov (1986) in defining IFS is interesting and useful. Fuzzy sets are IFS but the converse is not necessarily true Atanassov (1986). In fact there are situation where IFS theory is more appropriate to deal with (De, S.K., 2001). Besides, it has been cultured in (H. Bustince, P.Burillo, 1996) that vague sets (D.Doubois, H.Prade, 1980) are nothing but IFS.

The notion of Interval-Valued fuzzy sets was first introduced by Zadeh [1] as an extension of fuzzy sets in which the values of the membership degrees are intervals of numbers instead of the numbers. Thus, interval-Valued fuzzy sets provide a more adequate description of uncertainty than the traditional fuzzy sets. It is therefore important to use interval-Valued fuzzy sets in applications. One of the main applications is in fuzzy control and the most computationally intensive part of fuzzy control is defuzzification. Since the transition of Interval-Valued fuzzy sets usually increases the amount of computations, it is vitally important to design some faster algorithms for the necessarily defuzzification. Since these sets are widely studied and used, it

is worth pointing out the works of Gorzalczany on approximate reasoning [2, 3], Roy and Biswas on Medical diagnosis [4] and Turksen on multivalued logic [5]. Many variants of these mathematical objects exist, under various names. One popular variant proposed by Atanassov starts by the specification of membership and non-membership degrees. Atanassov introduced the idea of defining a fuzzy set by ascribing a membership degree and a non-membership degree separately in such a way that sum of the two degrees must not exceed one. Such a pair was given the name of Intuitionistic Fuzzy Sets [6]. Atanassov and Gargov [7, 8] introduced the notion of Interval-Valued Intuitionistic Fuzzy Sets which is a generalization of both Intuitionistic Fuzzy Sets and Interval-Valued Fuzzy Sets.

In this paper, it is proposed a new technique to diagnose the symptom of the disease using Interval-Valued Intuitionistic Fuzzy Sets (IVIFS), the membership and non-membership values are not always possible up to our satisfaction, but in deterministic (hesitation) part has more important role here, the fact that in decision making, particularly in case of medical diagnosis, there is a fair chance of the existence of a non-zero hesitation part at each moment of evaluation.

II. PRELIMINARIES

In this section, some basic definitions and results needed and notations are given.

Definition 2.1. Let X be a non empty set.

Then a fuzzy set (FS for short) A is a set having the form

$$A = \left\{ (x, \mu_A(x)) : x \in X \right\}$$

where the function $\mu_A : X \rightarrow [0,1]$ is called the membership function and $\mu_A(x)$ is called the degree of membership of each element $x \in X$.

After the introduction of the concept of fuzzy set by Zadeh [9], several researchers were conducted on the generalization of the notion of a fuzzy set. The idea of "intuitionistic fuzzy set" was first published by Atanassov [6].

Definition 2.2: Let a set X be fixed. An intuitionistic fuzzy set (IFS) A in X is an object having the form

$$A = \left\{ (x, \mu_A(x), \gamma_A(x)) : x \in X \right\}, \text{ where the functions}$$

$\mu_A : X \rightarrow [0,1]$ and $\gamma_A : X \rightarrow [0,1]$ define the degree of membership and degree of non-membership respectively of the element $x \in X$ to the set A, which is a subset of X, and for every $x \in X$, $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$.

The amount $\alpha_A(x) = 1 - \mu_A(x) - \gamma_A(x)$ is called thehesitation part, which may cater to either membership value or non-membership value or both [10].

Definition 2.3: Let X and Y are two sets.

An intuitionistic fuzzy relation (IFR) R from X to Y is an IFS of $X \times Y$ characterized by the membership function μ_R and non-membership function γ_R . An IFR R from X to Y will be denoted by $R(X \rightarrow Y)$ [10].

Definition 2.4: Let $Q(X \rightarrow Y)$ and $R(Y \rightarrow Z)$ be two IFRs.

The max-min-max composition $R \circ Q$ is the intuitionistic fuzzy relation from X to Z, defined by the membership function

$$\mu_{R \circ Q}(x, z) = \max [\mu_Q(x, y) \wedge \mu_R(y, z)]$$

and the non-membership function

$$\gamma_{R \circ Q}(x, z) = \min [\gamma_Q(x, y) \vee \gamma_R(y, z)]$$

$$\forall (x, z) \in X \times Z \text{ and } \forall y \in Y \text{ [10].}$$

Definition 2.5: An Interval-valued Intuitionistic Fuzzy Set A over an universe set X is defined as the object of the form

$$A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \},$$

where $\mu_A : X \rightarrow \text{Int}[0,1]$ and $\gamma_A : X \rightarrow \text{Int}[0,1]$ are functions such that the condition:

$$\forall x \in X, \sup \mu_A(x) + \sup \gamma_A(x) \leq 1 \text{ issatisfied (where}$$

$\text{Int}([0,1])$ is the set of all closed sub-intervals of $[0,1]$). The class of all interval valued intuitionistic fuzzy sets on X is denoted by $\text{IVIFS}(X)$ [7].

Definition 2.6. Let $A, B \in \text{IVIFS}(X)$, then

some operations can be defined as follows:

$$A \cup B = \{ \langle x, [\mu_{AL}(x) \vee \mu_{BL}(x), \mu_{AU}(x) \vee \mu_{BU}(x)], [\gamma_{AL}(x) \wedge \gamma_{BL}(x), \gamma_{AU}(x) \wedge \gamma_{BU}(x)] \rangle : x \in X \}$$

$$A \cap B = \{ \langle x, [\mu_{AL}(x) \wedge \mu_{BL}(x), \mu_{AU}(x) \wedge \mu_{BU}(x)], [\gamma_{AL}(x) \vee \gamma_{BL}(x), \gamma_{AU}(x) \vee \gamma_{BU}(x)] \rangle : x \in X \}$$

Where

\vee, \wedge stand for max and min operators, respectively and AL, BL, AU and BU stands for lower value of A, lower value of B, upper value of A, upper value of B, respectively [6].

Definition 2.7. The operator

$$\oplus_{\alpha} A = \{ \langle x, \alpha_A(x) \times \mu_A(x) + \mathbf{1} - \alpha_A(x), \alpha_A(x) \times \gamma_A(x) : x \in X \rangle \}$$

Where $\alpha_A(x)$ be the hesitation part (i.e., $\alpha_A(x) = \mathbf{1} - \mu_A(x) - \gamma_A(x)$) which may cater to either membership value or non-membership value or both [11].

1. Medical Diagnosis

Suppose S is a set of symptoms, D is a set of diagnosis and P is a set of patients. Let M_1 be an Interval-Valued Intuitionistic

Fuzzy Relation $M_1(P \rightarrow S)$ and M_2 from the set of patients to the set of symptoms S, i.e., $M_2(S \rightarrow D)$ then

$$Y_1 = M_1 \cup M_2 = \{ \langle x, [\mu_{M_1L}(x) \vee \mu_{M_2L}(x), \mu_{M_1U}(x) \vee \mu_{M_2U}(x)], [\gamma_{M_1L}(x) \wedge \gamma_{M_2L}(x), \gamma_{M_1U}(x) \wedge \gamma_{M_2U}(x)] \rangle : x \in X \}$$

$$Y_2 = M_1 \cap M_2 = \{ \langle x, [\mu_{M_1L}(x) \wedge \mu_{M_2L}(x), \mu_{M_1U}(x) \wedge \mu_{M_2U}(x)], [\gamma_{M_1L}(x) \vee \gamma_{M_2L}(x), \gamma_{M_1U}(x) \vee \gamma_{M_2U}(x)] \rangle : x \in X \}$$

Where

$$M_1 = (m_{ij})_{mn} = \langle [\mu_{mijL}, \mu_{mijU}], [\gamma_{mijU}, \gamma_{mijL}] \rangle \text{ i} = 1, \dots, m \text{ and } \text{j} = 1, \dots, n.$$

$$M_2 = (m_{jk})_{np} = \langle [\mu_{m_2jkL}, \mu_{m_2jkU}], [\gamma_{m_2jkU}, \gamma_{m_2jkL}] \rangle \text{ j} = 1, \dots, n \text{ and } \text{k} = 1, \dots, p.$$

$$Y_3 = \oplus_{\alpha} M_3 = \{ \alpha_{M_3}(x) \times \mu_{M_3U}(x) + \mathbf{1} - \alpha_{M_3}(x), \alpha_{M_3}(x) \times \gamma_{M_3U}(x) \}$$

$$\alpha_{M_3}(x) = \mathbf{1} - \mu_{M_3U}(x) - \gamma_{M_3U}(x) \text{ and}$$

$$Y_4 = \mu_Q(x) \wedge \gamma_Q(x) = \min \{ \mu_Q(x), \gamma_Q(x) \}.$$

III. ALGORITHM

Step1: Compute $M_1 \circ M_2 = M_3$ (using Y_1 and Y_2).

Step2: M_3 values are applied in Y_3 and the result is named as Q.

Step3: Q values are applied in Y_4 and the result is named as T.

Step4: Finally, we selected the minimum value from each row of T and then we conclude that the patient p_i is suffering from the disease d_r .

2. Case study

Let there be four patients Case1, Case2, Case3 and Case4 in a hospital. Their symptoms are temperature, headache, stomachpain, cough. Clearly,

$P = \{Case1, Case2, Case3, Case4\}$ and the set of symptoms

$S = \{Temperature; headahe; stomach pain; cough\}$. Let the set of diagnosis be

$D = \{Viral Fever; Malaria; Typhoid; Stomach problem\}$

The interval-valued intuitionistic fuzzy relation $M_1 (P \rightarrow S)$ is given as in Table1.

Table1[12]

M_1	Temperature	Headache	Stomach pain	Cough
Case1	$\langle [.6, .8], [.02, .1] \rangle$	$\langle [.4, .6], [.02, .1] \rangle$		$\langle [.4, .6], [.02, .1] \rangle$
Case2	$\langle [0, 0], [.6, .8] \rangle$	$\langle [.2, .4], [.2, .4] \rangle$	$\langle [.1, .2], [.6, .8] \rangle$	$\langle [.02, .1], [.5, .7] \rangle$
Case3	$\langle [.6, .8], [.02, .1] \rangle$	$\langle [.6, .8], [.02, .1] \rangle$	$\langle [0, 0], [.4, .6] \rangle$	$\langle [.1, .2], [.5, .7] \rangle$
Case4	$\langle [.4, .6], [.02, .1] \rangle$	$\langle [.3, .5], [.2, .4] \rangle$	$\langle [.1, .3], [.2, .4] \rangle$	$\langle [.5, .7], [.1, .2] \rangle$

The interval-valued intuitionistic fuzzy relation $M_2 (S \rightarrow D)$ is given as in Table2.

Table2[12]

M_2	Viral fever	Malaria	Typhoid	Stomach problem
Temperature	$\langle [.5, .4], [.02, .1] \rangle$	$\langle [.1, .7], [.02, .1] \rangle$	$\langle [.4, .6], [.02, .1] \rangle$	$\langle [.4, .1], [.02, .7] \rangle$
Headache	$\langle [.0, .3], [.5, .5] \rangle$	$\langle [.02, .2], [.2, .6] \rangle$	$\langle [.1, .3], [.2, .3] \rangle$	$\langle [.02, .2], [.3, .4] \rangle$
Stomach pain	$\langle [.06, .1], [.02, .7] \rangle$	$\langle [0, 0], [.02, .9] \rangle$	$\langle [.0, .2], [.3, .7] \rangle$	$\langle [.1, .8], [.0, .0] \rangle$
Cough	$\langle [.2, .4], [.02, .3] \rangle$	$\langle [.3, .7], [0, 0] \rangle$	$\langle [.1, .2], [.2, .6] \rangle$	$\langle [.1, .2], [.1, .7] \rangle$

Table3

M_3	Viral fever	Malaria	Typhoid	Stomach problem
Case1	$\langle [.6, .4], [.02, .1] \rangle$	$\langle [0, .7], [.6, .1] \rangle$	$\langle [.6, .6], [.02, .1] \rangle$	$\langle [.4, .2], [.02, .4] \rangle$
Case2	$\langle [.4, .3], [.02, .5] \rangle$	$\langle [.2, .2], [.2, .6] \rangle$	$\langle [.6, .4], [.02, .4] \rangle$	$\langle [.3, .6], [.2, .1] \rangle$
Case3	$\langle [.1, .4], [.6, .1] \rangle$	$\langle [.4, .7], [.02, .1] \rangle$	$\langle [0, .6], [.4, .1] \rangle$	$\langle [.1, .2], [.2, .4] \rangle$
Case4	$\langle [.4, .4], [.02, .1] \rangle$	$\langle [.02, .7], [.5, .1] \rangle$	$\langle [.1, .5], [.5, .3] \rangle$	$\langle [.5, .3], [.1, .4] \rangle$

Table4

Q	Viral fever	Malaria	Typhoid	Stomach problem
Case1	[0.70, 0.05]	[0.94, 0.02]	[0.88, 0.03]	[0.68, 0.16]
Case2	[0.86, 0.10]	[0.84, 0.12]	[0.88, 0.08]	[0.88, 0.02]
Case3	[0.70, 0.05]	[0.94, 0.02]	[0.88, 0.03]	[0.68, 0.16]
	[0.70, 0.05]	[0.94, 0.02]	[0.90, 0.06]	[0.79, 0.12]

Table5

T	Viral fever	Malaria	Typhoid	Stomach problem
Case1	0.05	0.02	0.03	0.16
Case2	0.10	0.12	0.08	0.02
Case3	0.05	0.02	0.03	0.16
	0.05	0.02	0.06	0.12

The doctor agrees then Case 1, Case 3 and Case 4 suffer from Malaria whereas Case 2 faces Stomach problem.

IV. CONCLUSION

In this paper the new technique to diagnose the symptom of the disease using Interval-Valued Intuitionistic Fuzzy Set (IVIFS) with a logical operators is successful and effective, the logical operators plays a vital role as they deal with the hesitation. Integrated Intelligent Research (IIR) part also membership and non-membership values are not always up to our satisfaction. This new technique can be used in our modern times to solve many problems faced by patients.

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