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A Modified Algorithm for Solving Interval Valued Fuzzy Relational Equations

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Abstract - In this paper, it is introduced a modified algorithm is proposed to solve the Interval valued (I – V) fuzzy relational equation P·Q = R with max-min composition and max productcomposition. The algorithm operates systematically and graphically on a matrix pattern to get all the interval solution of P. An example is given to illustrate its effectiveness.

Keywords: Interval valued fuzzy matrices, Fuzzy relational equations, Interval valued fuzzy relational equations, Interval maximum solution, Algorithms.

I. INTRODUCTION

Thomason has introduced the concept of fuzzy matrices [2]. After that a lot of works have been done on fuzzy matrices and its variants [3, 4, 5]. It is well known that the membership value completely depends on the decision maker's, its habit, mentality etc. So, sometimes it happens that the membership value can not be measured as a point, but, it can be measured appropriately as an interval. Sometimes, the measurement becomes impossible due to the rapid variation of the characteristics of the system whose membership values are to be determined. It is well known that the fuzzy relational equation xA = b, where A is a fuzzy matrix andb is a fuzzy vector is consistent, then it has a unique maximum solution and a finite number of minimum solutions [6]. A study on fuzzy relational equations was introduced by Sanchez [7] and later developed by many researchers, for more details one may refer [8]. In [9], Kim and Roush have developed a theory of fuzzy matricesanalogous to that for Boolean Matrices [1]. In [10], Cho has proved that a fuzzy relationalequation xA = b is consistent whenever A is regular and bX is a solution for some g-inverse X of A.

Recently the concept of Interval valued fuzzy matrices as a generalization of fuzzy matrix was introduced and developed by Shyamal and Pal [11], by extending the max-min operations on fuzzy algebra F = [0, 1], for elements a, b $\in$ F determined as a + b = max{a, b} and a.b = min{a, b} and the standard order '$\geq$' of real numbers over F. Let A be an element of F_{mn} .A is regular in F_{mn} if AXA=A for some X $\in F_{mn}$of A. Such X is called a generalized inverse of A and is denoted by A^{-} [10]. A new algorithm is proposed to solve the fuzzy relation equation P ∘ Q = R with max–min composition and max–product composition in [12].In [14], consistency of system of Interval fuzzy relational equation x A^I = $b^I$where A^I is the set of fuzzy matrices { A' / $A_L \leq$ A' $\leq A_U$} and b^I= {b' / $b_L \leq$ b' $\leq b_U$} isdiscussed and complete set of solutions for $b_L \leq$ xA' $\leq b_U$ is determined. Recently, it was restructured by G Li and SC Fang in [15]. In [16], consistency of the Interval valued fuzzyrelational equations and the complete set of solutions of xA = b where A is an Intervalvalued fuzzy matrix and b is an interval valued vector is determined and Equivalentcondition for the existence of Interval maximum solution is obtained. In this paper, a modified algorithm is proposed to solve the Interval valued fuzzy relational equations as a generalization of that of fuzzy relational equations. In section 2, we present the basic definitions and notations, required results on fuzzy relational equations. In section 3, we have proposed a modified algorithm to solve the Interval valued *(I – V)* fuzzy relational equation *P·Q = R* with *max-min* composition and max product composition. The algorithm operates systematically and graphically on a matrix pattern to get all the interval solution of *P*. An example is given to illustrate its effectiveness.

II. PRELIMINARIES

In this section, some basic definitions and results needed and notations are given. Let$(IVFM)_{mn}$, F_{mn}, F_m, N_n denotes the set of all m×n interval valued fuzzy matrices, set ofall fuzzy matrices, set of all fuzzy vectors and set of all natural numbers 1 to nrespectively.

Let I = {1,2,...,m} and J = { 1,2, …,n} be index set. Also define X = { x $\in F_m$ /0$\leq x_i \leq$ 1, for all i $\in$ I} and Ω (A,b) = {x $\in$ X / xA = b} which represents the solution setof system xA = b where A $\in F_{mn}$.

According to [8, 7] when Ω(A,b) ≠ φ for some fuzzy matrix A, then it can becompletely determined by the unique maximum solution and a finite number of minimumsolutions. Moreover, $\hat{x} \in \Omega(A,b)$ is called the maximum solution, if x $\leq \hat{x}$ for all x $\in \Omega(A,b)$ and $\check{x} \in \Omega(A,b)$ is called the minimum solution, if $\check{x} \leq$ x for all x $\in \Omega(A,b)$. IfΩ(A,b) ≠ φ, then the unique maximum solution of the equation xA = b is determined by the following formula [7]

$$\hat{x} = A_U \circ b_U = \left(min\left(a_{ijU} \circ b_{jU} \right) \right), j \in J, i \in I \qquad (2.0)$$

where $a_{ijU} \circ b_{jU} = \begin{cases} 1, & if\ a_{ijU} \leq b_{jU} \\ b_{jU}, & otherwise \end{cases}$

Definition 2.1.Interval-valued fuzzy matrix (IVFM).In[13] , An interval-valued fuzzy matrix of order m×n is defined as, A = $(a_{ij})_{m \times n}$, where a_{ij}=[a_{ijL}, a_{ijU}] is a subinterval of the interval [0, 1], as the Interval matrix A = [A_L, A_U] whose ij^{th} entry is the interval [a_{ijL} , a_{ijU}]. Hence the lower limit A_L = (a_{ijL}) and upper limit A_U = (a_{ijU}) are fuzzy matrices such that $A_L \leq A_U$.
By using the IVFM Matrix operation introduced and developed in [11], we have discussed the regularity of IVFM in terms of the regularity of the lower and upper limit Matrices A_L and A_U [13].is the ij^{th} element of A, represents the membership value.

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All the elements of an IVFM are intervals and all the intervals are the subintervals of the interval [0, 1].

Example 2.2. In [17], For example, we consider a network $N = (V,E)$ consisting n nodes (locations of city) and m edges (roads) connecting the location of a city. If we measure the crowdof theroads of the network for particular time duration, it is quite impossibleto measure the crowd as a single value, because, the crowd in aduration is not fixed, it varies timeto time. So, moreconvenient technique to grade the crowd is aninterval not a point. In this case, thenetwork N is called interval valuedfuzzy network and the correspondingmatrix (representing the crowd)is called interval valued fuzzy matrix.Let $A_N^{(T)}$ be the interval valued fuzzymatrix corresponding to the network$N$, representing the crowd of Nduring time interval T. The ij^{th}element $a_{ij}$of $A_N^{(T)}$ is defined as

$$a_{ij} = \begin{cases} [0,0], & if \ i = j, \\ [a_{L,} a_U], & if \ (i,j) \in E, \\ [1,1], & if \ (i,j) \notin E. \end{cases}$$

Where a_L and a_U are the lower and upper limits of the crowd of the road connecting the locations of city $i$and j.

[0.5, 0.7]

[0.8, 0.9]
[0.8, 1]

[.6, 0.8] [0.7, 0.8]
[0.7,1]

[0.8, 0.9]

[0.8, 1]

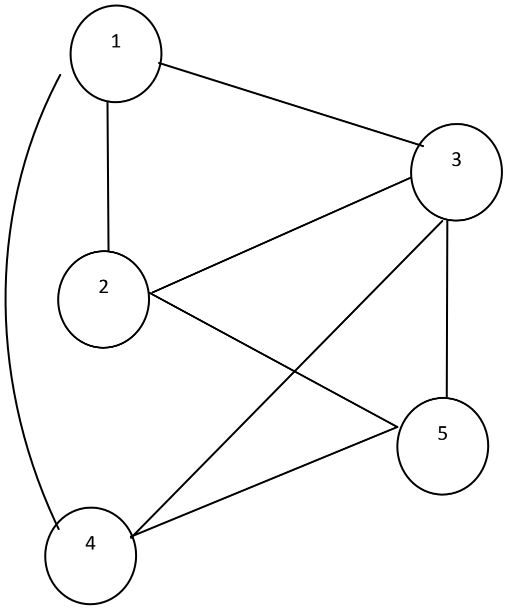

Fig 1: A network

City- Puducherry
1- Jipmer Hospital
2- Nehru Street
3- Pondicherry University
4- Rajiv Gandhi Hospital
5- Beach

For illustration, we consider a network shown in Figure 1 containing 5 nodes (location of city) and 8 edges (roads). The numbers adjacent to the edges represent the crowd of the roads connecting the corresponding city. The matrixrepresentation of crowd of the network of Figure 1 during the time interval T is shown below:

$$\begin{bmatrix} [0,0] & [.8,.9] & [.5,.7] & [.6,.8] & [1,\ 1] \\ [.8,.9] & [0,0] & [0.8,1] & [1,\ 1] & [.7,1] \\ [.5,.7] & [.8,1] & [0,\ 0] & [.8,.9] & [.7,.8] \\ [.6,.8] & [1,1] & [.8,.9] & [0,\ 0] & [.8,1] \\ [1,1] & [.7,1] & [.7,.8] & [.8,1] & [0,0] \end{bmatrix}$$

Definition 2.3. For a pair of fuzzy matrices $E = (e_{ij})$ and $F = (f_{ij})$ in F_{mn} such that $E \leq F$,let us define the interval matrix denoted as [E,F], whose ij^{th} entry is the interval with lower limit e_{ij} and upper limit f_{ij}, that is $[e_{ij}, f_{ij}]$.

In particular for E = F, IVFM [E, E] reduces to $E \in F_{mn}$
For $A = (a_{ij}) = ([a_{ijL}, a_{ijU}]) \in (IVFM)_{mn}$ let us define $A_L = (a_{ijL})$ and $A_U = (a_{ijU})$ clearly A_L and A_U belongs to F_{mn} such that $A_L \leq A_U$. Therefore an IVFM A can be written as $A = [A_L, A_U]$(2.1)
Here we shall follow the basic operations on IVFM as given in [13].

For $A = (a_{ij})_{mn} = ([a_{ijL}, a_{ijU}])$ and $B = (b_{ij})_{np} = (b_{ijL}, b_{ijU}])$ their product denoted as$AB = (C_{ij}) =$

$$\left[\sum_{k=1}^{n} a_{ik} b_{kj}\right] \quad i = 1,\ 2, 3, \ldots.., m \ and \ j = 1, 2, \ldots., p$$

$$= \left[\sum_{k=1}^{n} (a_{ikL}, b_{kjL}), \sum_{k=1}^{n} (a_{ikU}, b_{kjU})\right]$$

If $A = [A_L, A_U]$ and $B = [B_L, B_U]$ then
$AB = [A_L B_L, A_U B_U]$(2.2)

For $A = (a_{ij}), B = (b_{ij}) \in (IVFM)_{mn}$

$A \leq B$ if and only if $a_{ijL} \leq b_{ijL}$ and $a_{ijU} \leq b_{ijU}$for all i =1,...m and j = 1,2,…n (2.3). In particular if $a_{ijL} = a_{ijU}$ and $b_{ijL} = b_{ijU}$ then (2.2) reduces to the standard max-min composition of fuzzy matrices [9, 8]
Lemma 2.4.For $A = [A_L, A_U] \in (IVFM)_{mn}$and $B = [B_L, B_U] \in (IVFM)_{np}$, the following hold.

(i) $AT = [A_L^T, A_U^T]$
(ii) $AB = [A_L B_L, A_U B_U]$ [13].

Lemma 2.5.Let $A = [A_L, A_U] \in (IVFM)_{mn}$, then A is regularIVFM $\Leftrightarrow$ A_L and $A_U \in F_{mn}$ are regular [13].

Remark 2.6.If A is regular IVFM then there exist an IVFM X such that AXA = A, X iscalled g–inverse of A and X $\in A\{1\}$ the set of all g–inverses of A. By (2.2), AXA = Areduces to $A_L X_L A_L = A_L$and $A_U X_U A_U = A_U$for some $X_L \in A_L\{1\}$ and $X_U \in A_U\{1\}$. Therefore $X = [X_L, X_U]$ is a g–inverse of A.

Lemma 2.7.Let x A = b where $A = (a_{ij})F_{mn}$, $b = (b_{ij}) \in F_{1n}$. If$\max_j (a_{ij}) < b_k$ for some $k \in N_n$, then $\Omega(A,b) = \varphi$ [8].

In [15], the set of solution for xA' = b'where $A' \in A^I = \{ A'/ A' \in [A_L, A_U]\}$ and $b' \in b^I = \{ b'/ b_L \le b' \le b_U\}$2.4) is determine further, when the solution set is not empty, it is shown that xA' = b' has onemaximum solution and a finite number of minimum solutions.

Lemma 2.8.x is a solution of xA' = b' where $A' \in A^I = \{ A'/ A' \in [A_L,A_U]\}$ and $b' \in b^I = \{ b'/ b' \in [b_L,b_U]\}$ if and only if $x \in x^I$ for some $x^I \in \Omega(A^I, b^I)$where $x^I = \{ x'/ x_L \le x' \le x_U\}$.

Lemma 2.9.For the equation $xA= b$ where $x = [x_j / j \in N_m]$, $b = [b_k / k \in N_n]$ and $A \in F_{mn}$,

$\Omega(A, b) \neq \varphi$, if and only if $\hat{x} = [\hat{x}_j / j \in N_m]$, defined as $\hat{x}_j = \min \sigma(a_{ik}, b_k)$ where $\sigma(a_{ik}, b_k) = \begin{cases} b_k, & \text{if } a_{ik} > b_k \\ 1, & \text{otherwise} \end{cases}$

III. A MODIFIED ALGORITHM FOR SOLVING *I – V* FUZZY RELATIONAL EQUATIONS

In this section, modified algorithm is proposed to solve the Interval valued *(I – V)* fuzzyrelational equation $P \cdot Q = R$ with *max-min* composition and max product composition.The algorithm operates systematically and graphically on a matrix pattern to get all theinterval solution P *in* the *(I –V)*fuzzy relational equation $P.Q = R$. An example is given to illustrate its effectiveness.

Let interval valued Fuzzy matrix$Q = [q_{jk}]_{m\times n}$ be called the state matrix and Interval valued fuzzy vector $r = [r_k]$ be called the output vector with $q_{jk} \in [0, 1]$, $r_k = [0,1]$ for all $j \in J$and $k \in K$, where $J=\{1,2,....m\}$ and $K= \{1,2,.....,n\}$, $m,n \in N$. The problem is to determine all vectors $p \in P = \{ p = [p_j]_{1\times m} / p_j \in [0, 1]\}$ satisfying Interval valued fuzzy relational equation.

$$p \cdot Q = r \quad (3.1)$$

Then by using Lemma 2.2, $p \cdot Q = r$ can be expressed as the following fuzzy relational equations

$$P_L \cdot Q_L = r_L (3.2) \text{ and } P_U \cdot Q_U = r_U (3.3)$$

where '·' denotes the *max-min* composition with $\max_{j\in J} \min(p_j, q_{jk}) = r_k$or *max-product*composition with $\max_{j\in J} (p_j, q_{jk}) = r_k$for all $k \in K$.

Lemma 3.1.The (I-V) fuzzy relational equation of the form (3.1) isconsistent ifand only if the corresponding fuzzy relational Eqs. (3.2) and (3.3) areconsistent.The following algorithm and theorems are the generalization of results found in [18].

Algorithm 3.2.

Following are the algorithms for solving Interval valued fuzzy relational Eq. (3.1) with*max-min* (or *max-product*) composition:

Step 1: Check the consistentof the Eq. (3.1) by using Lemma 3.1 andLemma2.7.

Step 2: Rank the elements of *r*withdecreasing order and find themaximum solution $\bar{p}$ byUsingLemma 2.9 and [7].

Step3:Build the table $M=[m_{jk}], j=1,2,....,m, k=1,2,....,n.$ where $m_{jk} = (\bar{p}_j, q_{jk})$.Let$M = [M_L, M_U]$.

Therefore $m_{jk} = [m_{jkL}, m_{jkU}]$.Hence $M_L = [m_{jkL}]$ and $M_U = [m_{jkU}]$.This interval matrix M is calledthe matrix pattern.

Step 4: Mark m_{ij}, which satisfies $\min(\bar{p}_j, q_{jk}) = r_k$(or $p_j \cdot q_{jk} = r_k$), and then let themarkedm_{jk}be denoted by $\bar{m}_{jk}$.

Step 5: If $k_1$is the smallest k in all markedm_{jk}, then set $\underline{p}_{j1}$ to be the smallerone of the twoelements in $\bar{m}_{j1k1}$ (orset $\underline{p}_{j1}$).

Step 6: Delete the j_1^{th} row and k_1^{th} columnof M.

Step 7:In all remained and marked $\bar{m}_{jk}$ findthesmallest k and choose $\underline{p}'_{j1}$ to bethe smaller one of the two elementsin $\bar{m}_{j2k2}$. If $\underline{p}_{j1} > \underline{p}'_{j1}$ thenset $\underline{p}_{j2}$ and Delete the j_2^{th} row and the k_2^{th} column of M.

Step 8:If it is not truedelete the k column.

Step 9: Repeat Step *7 & 8* until no markedm_{jk}remained.

Step 10: The other $\underline{p}_j$ which are not set insteps 5 to *8*, are set to be zero.

Lemma 3.3.If the interval valued fuzzy relational equation is of the form as (3.1), forgiven $m\times n$interval matrix Q and $1\times n$ interval vector r, the interval minimum solution $\underline{p}$ can be obtained by the above algorithm.

Proof:Owing the fact that the whole interval minimum solution of $\underline{p}$ can be derived stepby step from the above algorithm, we can prove it in a straight way.

The steps 1 and 2 are the standard procedures that has been illustrated in [13] for*(I-V)* fuzzy relational equation and in [7] for the corresponding fuzzy relational equations.Since $0 \le \underline{p}_j \le \bar{p}_j$, step 3 is in fact, to put all the possible interval solution elementstogether. That is, with step 3, we would not miss any possible interval solution in thesolving procedure. Thus, for deriving the whole minimum solutions $\underline{p}$, the procedure instep *3* is necessary. In step 4, according to step 3 and Lemma 2.9, we have the followingdeductions. That is after checking $\min(p_{jL}, q_{jkL}) = r_{kL}$and $\min(p_{jU}, q_{jkU}) = r_{kU}$, it asnoted that the position of$m_{jkL}$and m_{jkU}make $\min(\bar{p}_{jL}, q_{jkL}) = r_{kL}$ $\min(\bar{p}_{jU}, q_{jkU}) = r_{kU}$happen and the results in $\bar{m}_{jkL}$ and $\bar{m}_{jkU}$ must be an element of the minimum solutions.Therefore by Lemma

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2.5(ii), *min* ($\overline{p}_j$, q_{jk}) = r_k it is noted that the position of $\overline{m}_{jk}$ make *min* ($\overline{p}_j$, q_{jk}) = r_k happen and the results in $\overline{m}_{jk}$ must be an element of the minimum solutions. Accordingly, we should mark all these elements. In step 5, we need to check interval minimum solution from high rank. If it is true, by step 4, pick the minimum solution for the corresponding p_j. In step 6: obviously, since the interval minimum solution for j_1^{th} row of M (i.e. j_1^{th} element of $\underline{p}$) has been gotten. That is, the minimum solution for the j_1^{th} element of $\underline{p}$ would not be repeated. The analogous procedure in steps 7 to 9 is to guarantee running of the algorithm from the left to right and from the upper to the bottom in interval matrix M and to get the whole interval minimum solutions. In step 10, if we cannot find the interval minimum solution for p_j the zero must be a solution naturally, the analogous proof can be done also for the *max–product* composition.

Remark 3.4. For the interval valued fuzzy relational equation $P.Q=R$, where P is $s \times m$ matrix and R is $s \times n$ matrix, the solution of the problem is obtained by solving the problem (3.1) repeatedly for each of the s-rows of P and the corresponding rows of R. Since the algorithm are similar no matter on solving *max-min* or *max–product* composition we only present an example with *max-min* composition to illustrate the procedure of the algorithm. This is illustrated in the following:

Example 3.5. In [15], Consider an interval valued fuzzy relational equation $P \cdot Q = R$ with *max-min* composition, where

$$Q = \begin{pmatrix} [0.3, 0.7] & [0.5, 0.6] \\ [0.4, 0.9] & [0.1, 0.7] \\ [0.2, 0.5] & [0.8, 1.0] \end{pmatrix}$$

R=([0.4,0.8]   [0.5,0.7])

By our representation (2.1) we have,

$$Q_L = \begin{pmatrix} [0.3] & [0.5] \\ [0.4] & [0.1] \\ [0.2] & [0.8] \end{pmatrix} \quad Q_U = \begin{pmatrix} [0.7] & [0.6] \\ [0.9] & [0.7] \\ [0.5] & [1.0] \end{pmatrix}$$

R_L = [0.4   0.5] and
R_U = [0.8   0.7]

First our task is to find the minimal solution $\underline{p} \in r^{1\times 2}$ in Eq. (3.2).

Step 1: It is known that the solution p exists by Lemma 2.7.
Step 2: By Corollary 3.8 of [16], the maximum solution p = [*1 1 0.5*]
Step 3: Build M_L as

$$M_L = \begin{pmatrix} (1, 0.3) & (1, 0.5) \\ (1, 0.4) & (1, 0.1) \\ (0.5, 0.2) & (0.5, 0.8) \end{pmatrix}$$

Step 4: Underline those elements which satisfies *min* ($\overline{p}_j$, q_{jk}) = r_k

$$M_L = \begin{pmatrix} (1, 0.3) & \underline{(1, 0.5)} \\ \underline{(1, 0.4)} & (1, 0.1) \\ (0.5, 0.2) & (0.5, 0.8) \end{pmatrix}$$

Step 5: Set $\underline{p}_1$ =min (1, 0.5) = 0.5; note here $j = l$.

Step 6: Delete the 1st and the 2nd column of M_L.

$$\cdot M_L = \begin{pmatrix} (1, 0.3) & \underline{(1, 0.5)} \\ \underline{(1, 0.4)} & (1, 0.1) \\ (0.5, 0.2) & (0.5, 0.8) \end{pmatrix}$$

$$M_L = \begin{pmatrix} (1, 0.3) & \underline{(1, 0.5)} \\ \underline{(1, 0.4)} & (1, 0.1) \\ (0.5, 0.2) & (0.5, 0.8) \end{pmatrix}$$

Step7: Choose $\underline{p}_1^{'}$ = min (1, 0.4) = 0.4 and check $\underline{p}_1 > \underline{p}_1^{'} \Rightarrow 0.5 > 0.4$. Condition is true. Therefore, set $\underline{p}_2$ =0.4; note here j=2. Step8: Delete the 2st row and the 1nd column of M_L.

Step 9: Until now, we have set $\underline{p}_1$ = 0.5 and $\underline{p}_2$ = 0.4 then the other $\underline{p}_j$ are set to be zero, that is $\underline{p}_3$ = *0*.

Therefore, we have only one minimal solution of Eq. (3.2) as $\underline{p}$ = [0.5   0.4   0]. Similarly, we have found only one minimal solution of Eq. (3.3) as $\underline{p}$ = [0.8   0.7   0].

Therefore, by Lemma 2.4(ii) we have minimal solution of the interval valued fuzzy relational equation $P \cdot Q = R$ as $\underline{p}$ = ([0.5, 0.8] [0.4, 0.7] [0, 0]).

General Solution of above Example 3.6 In [15], Consider an interval valued fuzzy relational equation $P \cdot Q = R$ with *max-min* composition, where

$$Q = \begin{pmatrix} [0.3, 0.7] & [0.5, 0.6] \\ [0.4, 0.9] & [0.1, 0.7] \\ [0.2, 0.5] & [0.8, 1.0] \end{pmatrix}$$

R= ([0.4, 0.8]   [0.5, 0.7])
By our representation (2.1) we have

$$Q_L = \begin{pmatrix} [0.3] & [0.5] \\ [0.4] & [0.1] \\ [0.2] & [0.8] \end{pmatrix} \quad Q_U = \begin{pmatrix} [0.7] & [0.6] \\ [0.9] & [0.7] \\ [0.5] & [1.0] \end{pmatrix}$$

R_L = [0.4   0.5] and
R_U = [0.8   0.7]

Step1: (Find the maximum solution)

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Compute

$$\hat{x} = A_U \circ b_U = \left(min\left(a_{ijU} \circ b_{jU} \right) \right), j \in J, i \in I$$

where $a_{ijU} \circ b_{jU} = \begin{cases} 1, & if\ a_{ijU} \le b_{jU} \\ b_{jU}, & otherwise \end{cases} \quad (using\ eq(2.0))$ Therefore,

$\hat{x}$ =(1, 0.8, 0.7).

Step2: (Check Feasibility)

If $\hat{x} Q_L \ge R_L$, continue. Otherwise,

stop! $X\left(\Omega\left(Q^I, R^I\right)\right) = \phi$,

Where $X\left(\Omega\left(Q^I, R^I\right)\right) = \bigcup_{x' \in \Omega\left(Q^I, R^I\right)} x^I$ [15].

Here, $\hat{x} Q_L \ge R_L$, $X\left(\Omega\left(Q^I, R^I\right)\right) \ne \phi$.

Step3: Compute

$$I_j = \left\{ i \in I / \min\left(\hat{x}_i, q_{ijL} \right) \ge r_{jL} \right\}, \forall j \in J.$$

We have $I_1\{2\},\ I_2 = \{1,3\}$.

Step4: Generate

$$\wedge = I_1 \times I_2 = \{(2,1), (2,3)\}$$ [15].

Step5: (Determine the minimum solutions) Foreach $f \in \wedge$, calculate F (f).

$$j_f^i = \left\{ j \in J / f_j = i \right\}, \forall i \in I, and$$

$$F : \wedge \to F_m \quad such\ that$$

$$\forall i \in I, F_i(f) = \begin{cases} \min_{j \in J_f^i} R_j, & if\ J_f^i \ne \phi \\ 0, & if\ J_f^i = \phi \end{cases}$$

Here, $f = (f_1, f_2) = (2,1) \in \wedge$ and (2, 3) $\in \wedge$.

Therefore, $j_f^1 = \{2\},\ j_f^2 = \{1\},\ j_f^3 = \{\ \}$ and $j_f^1 = \{\ \},\ j_f^2 = \{1\},\ j_f^3 = \{2\}$

Hence, $F(f)$ = [0.5 0.4 0] and [0 0.4 0.5].

Hence, we have two minimum solutions:

$\underline{x}_1$ = [0.5 0.40] and $\underline{x}_2$ = [0 0.4 0.5].

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